

FE - SEM II (CBBS)

11/2/18

A.M. - II

QP Code : 28611

(3 hours)

Total Marks: 80

- N.B. (1) Question No.1 is compulsory.  
 (2) Attempt any three questions out of the remaining five questions.  
 (3) Figures to right indicate full marks.

- Q.1
- (a) Prove that  $\int_0^1 \frac{dx}{\sqrt{-\log x}} = \sqrt{\pi}$  [3]
- (b) Solve  $\frac{d^3 y}{dx^3} - 5 \frac{d^2 y}{dx^2} + 8 \frac{dy}{dx} - 4y = 0$  [3]
- (c) Prove that  $\Delta \nabla = \nabla \Delta$  [3]
- (d) Solve  $[x y \sin(xy) + \cos(xy)] y dx + [x y \sin(xy) - \cos(xy)] x dy = 0$  [3]
- (e) Change to polar coordinates and evaluate  $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2 + y^2) dx dy$  [4]
- (f) Evaluate  $\int_0^1 \int_0^x (x^2 + y^2) x dy dx$  [4]
- Q.2
- (a) Solve  $(1+y^2) dx = (e^{\tan^{-1} y} - x) dy$  [6]
- (b) Change the order of integration and evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y + 1)\sqrt{1-x^2-y^2}} dy dx$  [6]
- (c) Prove that  $\int_0^\infty \frac{e^{-x} - e^{-ax}}{x \sec x} dx = \frac{1}{2} \log \left( \frac{a^2 + 1}{2} \right)$  [8]
- Q.3
- (a) Evaluate  $\int_1^e \int_1^y \int_1^x \log z dz dy dx$  [6]
- (b) Find the total area of the curve  $r = a \sin 2\theta$  [6]
- (c) Solve  $x^2 \frac{d^3 y}{dx^3} + 3x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = x^2 \log x$  [8]

[TURN OVER

-1-

FW-Con. 9032-16.

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Q.4

(a) Show that the length of the arc of the curve  $ay^2 = x^3$  from the origin to the

point whose abscissa is b is  $\frac{8a}{27} \left[ \left( 1 + \frac{9b}{4a} \right)^{3/2} - 1 \right]$  [6]

(b) Solve  $(D^2 - D - 2)y = 2 \log x + \frac{1}{x} + \frac{1}{x^2}$

(c) Apply Runge-kutta Method of fourth order to find an approximate value of  $y$  for  $\frac{dy}{dx} = x y$  with  $x_0 = 1, y_0 = -1$  at  $x = 1.2$  taking  $h = 0.1$

Q.5 (a) Solve  $(x^2 y - 2xy^2) dx - (x^3 - 3x^2 y) dy = 0$

(b) Using Taylor series Method obtain the solution of following differential equation

$\frac{dy}{dx} = 2y + 3e^x$  with  $y_0 = 0$  when  $x_0 = 0$  for  $x = 0.1, 0.2$  [6]

(c) Find the approximate value of  $\int_0^4 e^x dx$  [8]

by i) Trapezoidal Rule, ii) Simpson's 1/3<sup>rd</sup> Rule

Q.6 (a) In a circuit containing inductance  $L$ , resistance  $R$ , and voltage  $E$ , the current  $i$  is given by  $L \frac{di}{dt} + Ri = E$ . Find the current  $i$  at time  $t$  if at  $t = 0, i = 0$  and  $L, R, E$  are constants. [6]

(b) Evaluate  $\iint_R \frac{dx dy}{(1 + x^2 + y^2)^2}$  over one loop of the lemniscate [6]

$$(x^2 + y^2)^2 = x^2 - y^2$$

(c) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $z = 0$  and  $y + z = 4$  [8]

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