

S.E - IV Sem - Chem

1915116

Applied Maths - IV

SE/IV/CHM/CBGS/Am-IV  
Q.P. Code : 573002

(Hours)

(30)

[ Total Marks : 80 ]

- N.B. :**
- (1) Question No. ONE is compulsory.
  - (2) Solve any THREE Questions out of remaining FIVE.
  - (3) Figures to the right indicate full marks.
  - (4) Write the sub -questions of main question collectively together.

1. (a) Obtain the Fourier expansion of  $f(x) = \left(\frac{\pi-x}{2}\right)^2$  in the interval  $0 \leq x \leq 2\pi$  5
  - (b) Evaluate  $\int_C \bar{F} \cdot d\bar{r}$  where,  $\bar{F} = 3xi + (2xz - y)j + zk$  from  $(0, 0, 0)$  to  $(2, 1, 3)$  along the line joining the two points. 5
  - (c) Prove that  $f_1(x) = 1$ ,  $f_2(x) = x$ ,  $f_3(x) = (3x^2 - 1)/2$  are orthogonal over  $(-1, 1)$ . 5
  - (d) Find the Fourier Transform of  $f(x) = e^{-x^2/2}$  5
  2. (a) Evaluate by Green's Theorem  $\int_C (e^{2x} - xy^2)dx + (ye^x + y^2)dy$  where C is the closed curve bounded by  $y^2 = x$  and  $x^2 = y$ . 6
  - (b) Obtain half range sine series for  $f(x)$  when  $f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$  7
- Hence, Find the sum of  $\sum_{(2n-1)}^{\infty} \frac{1}{n^4}$
- (c) Determine the solution of one dimensional heat equation  $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$  under the boundary conditions  $u(0, t) = 0$ ,  $u(l, t) = 0$  and  $u(x, 0) = x$  ( $0 < x < l$ )  $l$  being the length of the rod. 7
  3. (a) Find the Fourier Integral representation of  $f(x) = \begin{cases} 0 & x < 0 \\ 1/2 & x = 0 \\ e^{-x} & x > 0 \end{cases}$  6
  - (b) Obtain complex form of Fourier Series for  $f(x) = \text{Cosh } 3x + \text{Sinh } 3x$  in  $(-3, 3)$  7

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- (c) Apply Stoke's Theorem to evaluate  $\int_C [(x+2y)dx + (x-z)dy + (y-z)dz]$  where 7

C is the boundary of the triangle with vertices (2,0,0) (0,3,0) and (0,0,6) oriented in the counter clockwise direction.

4. (a) Prove that  $\vec{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$  is a conservative field. Find the scalar potential  $\phi$  such that  $\nabla\phi = \vec{F}$ . Hence or otherwise find the work done by  $\vec{F}$  in displacing a particle from A(1,0,2) to B (0,1,1) along the straight line AB. 6

- (b) Find Fourier integral representation for muADDA.com 7

$$f(x) = \begin{cases} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

- (c) A rod of length 30 cms has its ends A and B kept at  $20^\circ\text{C}$  and  $80^\circ\text{C}$  respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to  $0^\circ\text{C}$  and kept  $80^\circ$ . Find the resulting temperature function  $u(x,t)$  taking  $x = 0$  at A. 7

5. (a) Find the Fourier Sine transform of  $\frac{e^{-ax}}{x}$  6

- (b) Find the Fourier Series for periodic function  $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$  7

- (c) Evaluate  $\iint_S \vec{F} \cdot d\vec{s}$  where  $\vec{F} = 4xi - 2y^2j + z^2k$  and S is the region bounded by  $y^2=4x$ ,  $x=1$ ,  $z=0$ ,  $z=3$  7

6. (a) Show that the set of functions  $\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), \dots$  is 6 orthogonal over  $(0, L)$

- (b) Find half range cosine series for  $f(x) = x$ ,  $0 < x < 2$  using parseval's identity 7

deduce that  $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$  muADDA.com

- (c) Evaluate the surface integral  $\iint_S (yzi - zxj + xyk) \cdot d\vec{s}$  where S is the surface of 7 the sphere  $x^2 + y^2 + z^2 = a^2$  in the first octant.