

AM-IV

SE/Chem/KT/Rev

Applied Maths

Q.P. Code : 12449

29/11/14

2.30 - 5.30

(3 Hours)

(29)

[ Total Marks : 80

- N.B. : (1) Question No.1 is compulsory. *Four (04)*  
 (2) Attempt any three out of the remaining ~~five~~ questions.  
 (3) Now programmable calculator is allowed.

1. (a) Find the Fourier's expansion of  $f(x) = x^2$ ,  $-\pi \leq x \leq \pi$  and hence, prove that 5

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

- (b) Evaluate  $\int_C \vec{F} \times d\vec{r}$  where  $\vec{F} = (2xy + z^2)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$  along the curve  $x = t, y = t^2, z = t^3$  from  $(0, 0, 0)$  to  $(1, 1, 1)$ . 5

- (c) Find the Fourier's Transform of  $f(x) = e^{-x^2/2}$ . 5

- (d) Find the circulation of  $\vec{F}$  round the curve  $C$  where  $\vec{F} = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$  and  $C$  is the circle  $x^2 + y^2 = a^2, z = 0$ . 5

2. (a) Obtain half range Sine series for  $f(x)$  when : 6

$$\vec{F}(x) = \begin{cases} x & 0 < x < (\pi/2) \\ \pi - x & (\pi/2) < x < \pi \end{cases}$$

Hence, find the sum of  $\sum_{n=1}^{\infty} \frac{1}{n^4}$

- (b) Show that the set of functions  $\{\sin x, \sin 3x, \sin 5x, \dots\}$   $n = 0, 1, 2, \dots$  is orthogonal over  $[0, \pi/2]$ . Hence construct orthonormal set of functions. 7

- (c) Find the area of the astroid  $x^{2/3} + y^{2/3} = a^{2/3}$  by using Green's Theorem. 7

3. (a) Find the Fourier cosine integral of the following function : 6

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$$

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(b) Prove that  $\int_A^B (2xy^3 - y^2 \cos x) dx + (1 - 2y \sin x + 3x^2 y^2) dy = \frac{\pi^2}{4}$  along arc  $2x = \pi y^2$  from  $A(0, 0)$  to  $B(\pi/2, 1)$  7

(c) Find the Fourier expansion of  $f(x) = 4 - x^2$  in the interval  $(0, 2)$ . Hence prove

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

4. (a) Obtain the complex form of Fourier series for  $f(x) = e^{ix}$  in  $(-1, 1)$  6

(b) Using Stoke's theorem formula  $\int_C x^2 dx + xy dy$  and  $C$  is boundary of the rectangle  $x=0, y=0, x=a, y=b$ . 7

(c) A tightly stretched string with fixed end point  $x=0$  and  $x=l$ , in the shape defined by  $y = Kx(l-x)$  where  $K$  is constant is released from this position of rest. Find 7

$$y(x, t) \text{ the vertical displacement if } \frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

5. (a) Using Gauss's Divergence theorem for  $\vec{F} = (4xi - 2y^2j + z^2k)$  taken over the region bounded by  $x^2 + y^2 = 4, z=0, z=3$ . 6

(b) Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  for  $0 < x < \pi, 0 < y < \pi$  with conditions given :  $u(0, y) = u(\pi, y) = u(x, \pi) = 0, u(x, 0) = \sin^2 x$  7

(c) Show that  $\vec{F} = (2xy + z^3)i + x^2j + 3z^2xk$  is a conservative field. Find its scalar potential and also work done in moving a particle from  $(1, -2, 1)$  to  $(3, 1, 4)$  7

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