

S.E. Sem III (CBAs) (Computer & I.T.)
A.M.-III

2011/12

Q.P. Code : 5067**(3 Hours)****[Total Marks : 80]****Instructions:**

- 1) Question No. 1 is compulsory.
- 2) Attempt any THREE of the remaining.
- 3) Figures to the right indicate full marks.

Q.1. A) Find Laplace of $\{t^5 \cos ht\}$ (5)

B) Find Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$ (5)

C) Find a, b, c, d, e if,

$f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic (5)

D) Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$ (5)

Q.2) A) If $f(z) = u + iv$ is analytic and $u + v = \frac{3 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$, find $f(z)$ (6)

B) Find inverse Z-transform of $f(z) = \frac{z+2}{z^2 - 2z + 1}$ for $|z| > 1$ (6)

C) Find Fourier series for $f(x) = \sqrt{1 - \cos x}$ in $(0, 2\pi)$

Hence, deduce that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$ (8)

Q.3) A) Find $L^{-1} \left\{ \frac{1}{(s-2)^4(s+3)} \right\}$ using Convolution theorem (6)

B) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = (3x^2 - 1)/2$ are orthogonal over $(-1, 1)$ (6)

C) Verify Green's theorem for $\int_C \bar{F} \cdot \bar{dr}$ where $\bar{F} = (x^2 - y^2)i + (x+y)j$ and C is the triangle with vertices $(0,0)$, $(1,1)$, $(2,1)$ (8)

[TURN OVER]

Q.4) A) Find Laplace Transform of $f(t) = |\sin pt|$, $t \geq 0$ (6)

B) Show that $\vec{F} = (ysinz - \sin x) i + (xsinz + 2yz) j + (xycosz + y^2) k$ is irrotational.

Hence, find its scalar potential. (6)

C) Obtain Fourier expansion of $f(x) = x + \frac{\pi}{2}$ where $-\pi < x < 0$

$$= \frac{\pi}{2} - x \text{ where } 0 < x < \pi$$

Hence, deduce that (i) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

$$(ii) \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \quad (8)$$

Q.5) A) Using Gauss Divergence theorem to evaluate $\iint_S \vec{N} \cdot \vec{F} ds$ where $\vec{F} = 4xi - 2y^2j + z^2k$
and S is the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$ (6)

B) Find $Z\{2^k \cos(3k+2)\}$, $k \geq 0$ muADDA.com (6)

C) Solve $(D^2 + 2D + 5)y = e^{-t} \sin t$, with $y(0) = 0$ and $y'(0) = 1$ (8)

Q.6) A) Find $L^{-1}\left\{\tan^{-1}\left(\frac{z}{s^2}\right)\right\}$ (6)

B) Find the bilinear transformation which maps the points 2, i, -2 onto points 1, i, -1 by using cross-ratio property. (6)

C) Find Fourier Sine integral representation for $f(x) = \frac{e^{-ax}}{x}$ (8)

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