

S.E. ETRX & EXTC Sem IV (CBGS)  
App. Maths -IV

27/11/14

QP Code 12440

(3 Hours)

[Total Marks : 80]

- N.B. : (1) Question No. 1 is **compulsory**.  
 (2) Solve any **three** questions from the remaining.

1. (a) Find the value of  $\mu$  which satisfy the equation.

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 $A^{100} x = \mu X$ , where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$$

- (b) Evaluate  $\int_0^{1+i} (x^2 + iy) dz$  along

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 $y = x$  and  $y = x^2$ .

- (c) Find the extremal of the function.

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$$\int_{x_1}^{x_2} [y^2 - y'^2 - 2y \cosh x] dx$$

- (d) Verify Cauchy-Schwartz inequality for the vectors.

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 $u = (-4, 2, 1)$  &  $v = (8, -4, -2)$ 

2. (a) Determine the function that gives the shortest distance between two given points.

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- (b) Find eigen values and eigen vectors of—

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$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

- (c) Obtain Taylor's and two distinct Laurent's series expansion of  $f(z) = \frac{z-1}{z^2-2z-3}$  about  $z = 0$  indicating the region of convergence.

**GN-Con. 8569-14.**

[ TURN OVER ]

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3. (a) Verify Caley-Hamilton theorem for

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ hence find } A^{-2}.$$

- (b) Evaluate by using Residue theorem.

$$\int_0^{2\pi} \frac{d\theta}{(2 + \cos\theta)^2}$$

- (c) Solve the boundary value problem.

$$I = \int_0^1 \left( 2xy - y^2 - y'^2 \right) dx$$

given  $y(0) = y(1) = 0$  by Rayleigh-Ritz method.

4. (a) Reduce the following Quadratic form

$$Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$$

into canonical form. Hence find its rank, index and signature.

- (b) Show that the matrix  $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$  is derogatory.

- (c) (i) Show that the set  $W = \{(1, x) | x \in \mathbb{R}\}$  is a subspace of  $\mathbb{R}^2$  under operations  $[1, x] + [1, y] = [1, x+y]$ ;  $k[1, x] = [1, kx]$ ;  $k$  is any scalar.  
(ii) Is the set  $W = \{[a, 1, 1] | a \in \mathbb{R}\}$  a subspace of  $\mathbb{R}^3$  under the usual addition and scalar multiplication?

5. (a) Find the plane curve of fixed Perimeter and maximum area.

- (b) Construct an orthonormal basis of  $\mathbb{R}^2$  by applying Gram Schmidt orthogonalization to  $S = \{[3, 1], [2, 2]\}$

- (c) Show that the matrix  $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalisable. Also find diagonal form and diagonalising matrix.

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6. (a) Evaluate  $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2 + 1)(x^2 + 4)} dx$  using Cauchy Residue Theorem. 6
- (b) If  $\phi(\alpha) = \oint_C \frac{ze^z}{z-\alpha} dz$  where c is  $|z - 2i| = 3$  6  
 find  $\phi(1), \phi'(2), \phi(3), \phi'(4)$
- (c) Show that the set V of positive real numbers with operations.  
 Addition :  $x + y = xy$   
 Scalar multiplication :  $kx = x^k$ .  
 is a vector space  
 where x, y are any two real numbers and k is any scalar.
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