EXTC & INST

SE/ Sem-III / App. Maths-III / EXTC & INST/ May 14

(3 Hours)   [Total Marks : 80

N. B. : (1) Question No. 1 (one) is **compulsory**.
      (2) Attempt any **3** (three) questions from the remaining questions.
      (3) Assume suitable data, if necessary.

1. (a) Evaluate $\displaystyle\int_0^\infty \frac{(\cos 6t - \cos 4t)}{t}\,dt$ 5

    (b) Obtain complex form of fourier series for $f(x) = e^{ax}$ in $(-1,1)$ 5

    (c) Find the work done in moving a particle in a force field given by   5
$\bar{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$ along the curve $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$.   muADDA.com

    (d) Find the orthogonal trajectory of the curves $3x^2 y + 2x^2 - y^3 - 2y^2 = \alpha$, where   5
α is a constant.

2. (a) Evaluate $\dfrac{d^2 y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin t$, $y(o) = 0$, $y'(o) = 0$, by Laplace transform   6

    (b) Show that $J_{5/2} = \sqrt{\dfrac{2}{\pi x}}\left[\dfrac{3 - x^2}{x^2}\sin x - \dfrac{3}{x}\cos x\right]$ 6

    (c) (i) Find the constants a, b, c so that   4
$\bar{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + (y + 2z)\hat{k}$ is irrotational.

      (ii) Prove that the angle between two surfaces $x^2 + y^2 + z^2 = 9$ and   4
$x^2 + y^2 - z = 3$ at the point $(2,-1,2)$ is $\cos^{-1}\left(\dfrac{8}{3\sqrt{21}}\right)$

3. (a) Obtain the fourier series of $f(x)$ given by   6
$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ x^2 & 0 \le x \le \pi \end{cases}$

    (b) Find the analytic function $f(z) = u + iv$ where $u = r^2\cos 2\theta - r\cos\theta + 2$ 6

    (c) Find Laplace transform of   muADDA.com   8
      (i) $te^{-3t}\cos 2t \cdot \cos 3t$

      (ii) $\dfrac{d}{dt}\left[\dfrac{\sin 3t}{t}\right]$

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4. (a) Evaluate $\int J_3(x)dx$ and Express the result in terms of J_0 and J_1　　6

   (b) Find half range sine series for　　6

   $f(x) = \pi x - x^2$ in $(0, \pi)$

   Hence deduce that $\dfrac{\pi^3}{32} = \dfrac{1}{12} - \dfrac{1}{3^2} + \dfrac{1}{5^2} - \dfrac{1}{7^2} + \dots$

   (c) Find inverse Laplace transform of　　8

   (i) $\dfrac{1}{s}\tanh^{-1}(s)$　　(ii) $\dfrac{se^{-2s}}{(s^2 + 2s + 2)}$

5. (a) Under the transformation $w + 2i = z + \dfrac{1}{z}$, show that the map of the circle $|z| =$　　6

   2 is an ellipse in w-plane.

   (b) Find half range cosine series of $f(x) = \sin x$ in $o \le x \le \pi$.　　6

   Hence deduce that

   $\dfrac{1}{1.3} + \dfrac{1}{3.5} + \dfrac{1}{5.7} + \dots = \dfrac{1}{2}$

   (c) Verify Green's theorem, for　　8

   $\oint_c (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where c is boundary of the region defined

   by x=0, y=0, and x+y = 1.

6. (a) Using convolution theorem, evaluate　　6

   $L^{-1}\left\{\dfrac{1}{(s-1)(s^2+4)}\right\}$

   (b) Find the bilinear transformation which maps the points　　6

   $z = 1, i, -1$ onto $w = 0, 1, \infty$

   (c) By using the appropriate theorem, Evaluate the following :-　　8

   (i) $\int \overline{F} \cdot d\overline{r}$ where $\overline{F} = (2x - y)\hat{i} - (yz^2)\hat{j} - (y^2z)\hat{k}$

   and c is the boundary of the upper half of the sphere $x^2 + y^2 + z^2 = 4$

   (ii) $\iint_s (9x\hat{i} + 6y\hat{j} - 10z\hat{k}) \cdot d\overline{s}$ where s is

   the surface of sphere with radius 2 units.

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