B.E VIII / Finite Element Analysis / MECH 14-05-15

(04 Hours) [Total Marks: 100]

N.B. (1) Question No:01 is compulsory
(2) Attempt any **FOUR** Questions from remaining **SIX** Questions.
(3) Assume suitable data where ever is necessary.
(4) Figures to the right indicate **full** marks.

Q.1 a) Define a bandwidth of a matrix? What is its significance in FEM? Calculate (10) the band width for the mentioned below F.E. mesh of One Dimensional field problem.
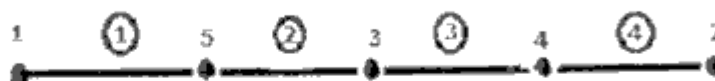

b) Find the weak form and its Quadratic functional for the following given (10) governing Differential Equation:

$$k\frac{d^2u}{dx^2} - \lambda u + 2x^2 = 0$$
where k, λ are constants, $0 < x < 1$

subject to $u(0) = 1$ and $u(1) = -2$

Q.2 a) Consider an axial tension problem given in Figure. The bar has a linearly (10) varying cross-sectional area $A = (x + 1)$ m 2 in the region $0\,m < x < 1\,m$. The Young's modulus is $E = 5 \times 10^7$ Pa. The bar is subjected to a point load $P = 200$ N at $x = 0.75$ m and The bar is constrained at $x = 0$ m.
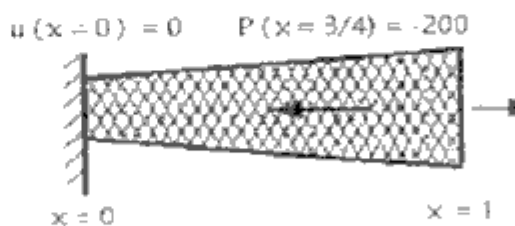
Construct the element stiffness matrix and force matrix. Solve the system of linear equations and find the nodal displacements and element stresses.
Suggest how to improve the finite element model to get more accurate results

RJ-Con. 8981-15. [TURN OVER

b) Linear Springs are connected to the carts as shown in figure. Imagine that only horizontal displacements are allowed. Write down the Global Equations and calculate the nodal displacements and reactions at the constraints. (10)
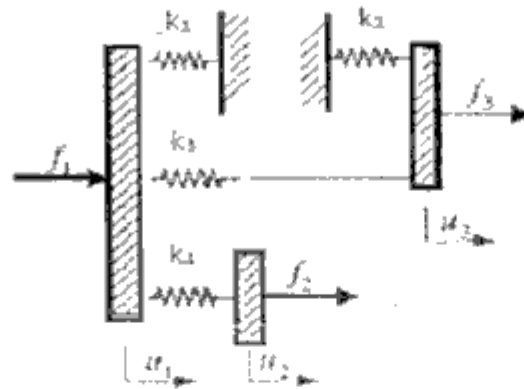
Q.3

a) Consider a three - noded element in one dimensional Heat Transfer application. The element length is 50 cms, with one of the node is located at x = 20 cms. The temperatures at the nodes are given by T_1 = 300 °C, T_2 = 100 °C, T_3 = 60 °C, , Find the temperature field at x = 3.5cms (8)

b) The following governing differential equation represents the flow of a Newtonian viscous fluid on an inclined flat surface. The momentum equation, for a fully developed steady laminar flow along the z coordinate, is given by (12)

$$-\mu \frac{d^2 w}{dx^2} = \rho g \cos\beta$$

Where w is the z component of the velocity, μ is the viscosity of the fluid, ρ is the density, g is the acceleration due to gravity, and β is the angle between the inclined surface and the vertical. The boundary conditions associated with the problem are that the shear stress is zero at x=0 and the velocity is zero at x = L.

$$\left(\frac{dw}{dx} \right)\bigg|_{x=0} = 0 \quad ; \quad w(L) = 0$$

Formulate the finite element equations (i e in terms of the stiffness matrix, unknown nodal field vector and load vectors) using Rayleigh Ritz's finite element method approach for a two-noded element of length L with the interpolation functions

[TURN OVER

RJ-Con. 8981-15.

Q.4 a)

Explain with the help of suitable sketches, about the connectivity conditions (05)
pertaining to Primary Variables and Secondary variables at junction nodes
during assembly.

b)

For the Three Bar plane Truss structure shown in figure. Determine the nodal (15)
displacements, stresses in each element and reaction at support by using
Multipoint constraint (MPC) method. Take $E = 207GPa$; $A = 5$ cm2 , P
$=103KN;L = 100cm$

For all members:
$E = 207$ GPa,
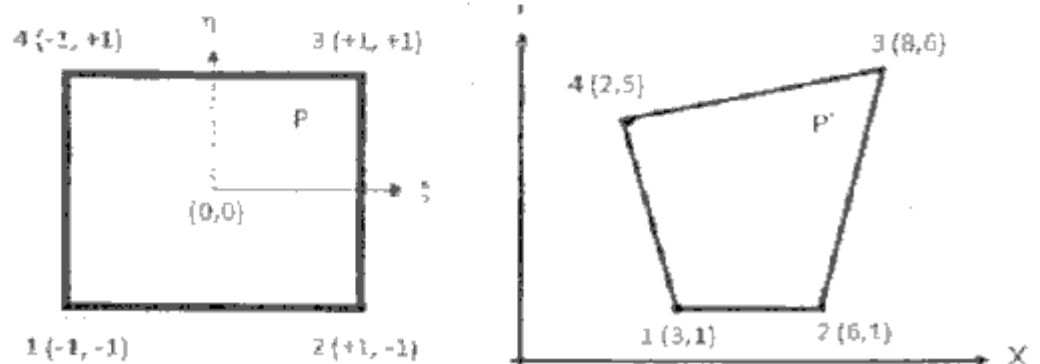$A = 5$ cm²

$\theta_3 = 45°$

$\beta = 60°$

P=1 000 kN
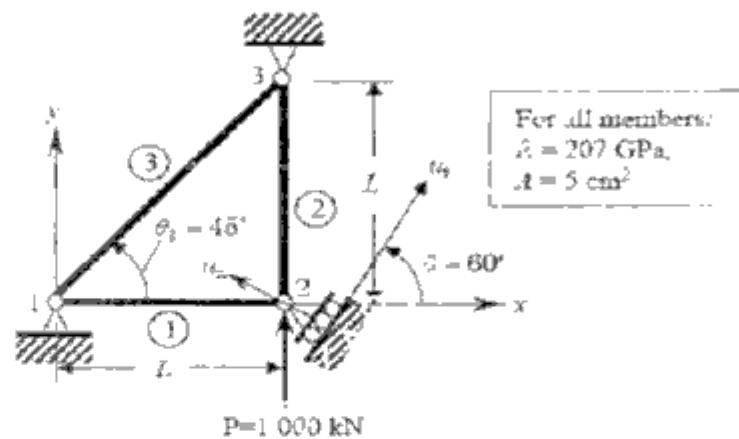
Q.5

a)

i) Explain Constant strain triangle (CST). (02)

ii) Derive 2-D quadratic interpolation shape functions for a triangular element in (08)
terms of Area Coordinates.

b) For the iso-parametric quadrilateral elements shown in Figure Determine
(a) Cartesion coordinates of the point P which has local coordinates ($\xi =$ (10)
0.57735 and $\eta = 0.57735$)

4 (-1, +1)   η   3 (+1, +1)

P

(0,0)

1 (-1, -1)   2 (+1, -1)

4 {2,5}   3 (8,6)

P'

1 (3,1)   2 (6,1)   X

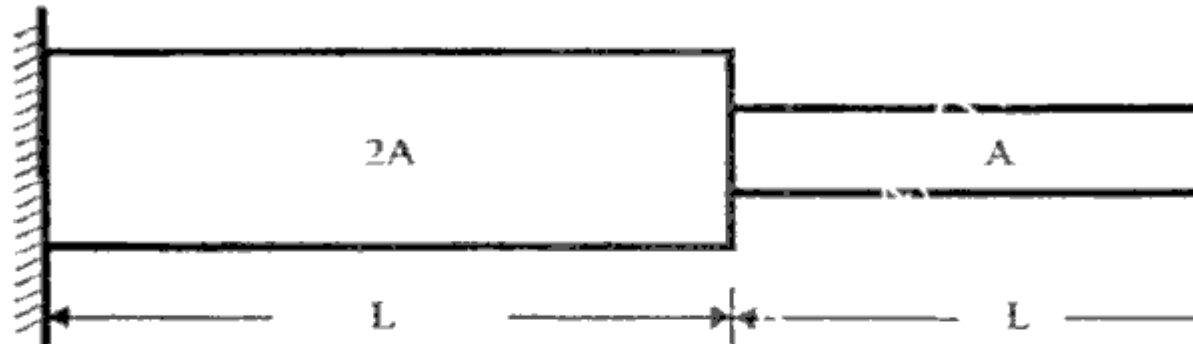**RJ-Con. 8981-15.**
[TURN OVER

Q.6 a) Discuss briefly higher order and iso-parametric elements with sketches

b) Find the natural frequencies of longitudinal vibrations of the con- stepped shaft of areas A and 2A and of equal lengths (L), as shown Compare the results obtained using lumped mass matrix approach

Q.7

a) Evaluate the following integral using Gaussian Legendre Quadrature for the integral and compare with exact result.

$$I = \int_0^3 \int_1^2 x\,y\,(1+x+y)\,dx\,dy$$

i	ξ_i	W_i
1	0.0	2.0
2	+ 0.5774	1.0
3	0.0	0.8889
	+ 0.7746	0.5556

b) Solve the following differential equation by using any of the Two Meth Galerkin ii) Least Squares iii) Sub Domain iv) Collocation

$$-\frac{d^2u}{dx^2} + 2x^2 = 0 ,\ 0 < x < 1;$$

Boundary Conditions are given: $u(0) = 0$, $u'(1) = 1$.
Compare your answers with exact at minimum three points within the d

RJ-Con. 8981-15.

COURSE: B.E. (SEM.VIII) (MECHANICAL ENGG.
COMMON WITH AUTOMOBILE ENGG.)

QP CODE: **8005**

Find Corrections are as follows:

Q. 2 a)	(Third Line in the problem) i) Young's Modulus $E = 5 \times 10^7$ Pa (In the figure) ii) Only consider single point load is acting at x=3/4 and cancel the arrow at extreme right side in the figure.
Q.3 a)	(Last line in the problem) Find the temperature field at x=35 cms instead of 3.5 cms
Q.4 b)	(Last Line in the problem) Load $P = 10^3$ KN (1000 KN)
Q.7 a)	Limits of the double integral are : 0 - 3 & 1 - 2

r	ξ_i	W_i
1	0.0	2.0
2	± 0.57774	1.0
3	0.0	0.8889
	± 0.7746	0.5556

Q.7 b)	$- \dfrac{d^2 u}{dx^2} + 2 x^2 = 0$