

sem. III / APP. Maths-III / MECH & PROD  
31-05-14

QP Code : NP-18610

(3 Hours)

( Total Marks : 80 )

- N.B. (1) Question no. 1 is compulsory.  
 (2) Solve any three questions out of the remaining Q.no. 2 to Q. no. 6.

1. (a) Find the inverse Laplace transform of

$$\frac{s^2 + 5}{(s^2 + 4s + 13)^2}$$

- (b) If  $V = 3x^2y + 6xy - y^3$ , show that the function V is harmonic, find the corresponding analytic function. muADDA.com

- (c) Evaluate  $\int_C \bar{z} dz$  where C is the upper half of the circle  $r = 1$ .

- (d) Prove that  $f_1(x) = 1$ ,  $f_2(x) = x$ ,  $f_3(x) = \frac{3x^2 - 1}{2}$  are orthogonal over  $(-1, 1)$ .

2. (a) Evaluate  $\int_0^\pi \frac{\cos at - \cos bt}{t} dt$

6

- (b) Obtain complex form of fourier series  $f(x) = e^{-ax}$  for in  $(-\pi, \pi)$

6

- (c) Using Crank-Nicholson simplified formula solve,  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$

8

$$u(0, t) = 0, u(4, t) = 0, u(x, 0) = \frac{x}{3}(16 - x^2)$$

Find  $u_{ij}$  for  $i = 0, 1, 2, 3, 4$ , and  $j = 0, 1, 2$ .

3. (a) Evaluate  $\int_C \frac{\sin^6 z}{(z - \pi/6)^3} dz$  where C is  $|z| = 1$  muADDA.com

6

- (b) Find the fourier expansion for  $f(x) = x - x^2$   $-1 < x < 1$

6

- (c) Determine the solution of one dimensional heat equation,  $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$  under the boundary conditions  $u(0, t) = 0$ ,  $u(\ell, t) = 0$  and  $u(x, 0) = x$ ,  $(0 < x < \ell)$ ,  $\ell$  being length of the rod.

8

[ TURN OVER ]

**Con. 9830-14.**

4. (a) Find inverse Laplace transform by using convolution theorem, 6

$$f(s) = \frac{s^2}{(s^2 - a^2)^2}$$

- (b) Find the image of the region bounded by  $x = 0, x = 2, y = 0, y = 2$  in the  $Z$  plane under transformation  $W = (1 + i) Z$ . 6

- (c) Find all possible Laurent's expansions of the function  $f(z) = \frac{7z-2}{z(z-2)(z+1)}$  about  $Z = -1$ . 8

5. (a) Solve  $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$  by Bender-Schmidt method, subject to the conditions 6

$u(0, t) = 0, u(x, 0) = 0, u(1, t) = t$  taking  $h = 0.25, 0 < x < 1$ .

- (b) Obtain half range sine series for  $f(x)$  when 6

$$f(x) = x, \quad 0 < x < \frac{\pi}{2}$$

$$= \pi - x, \quad \frac{\pi}{2} < x < \pi$$

- (c) Evaluate  $\int_0^\infty \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$  by using residues.  $a > 0, b > 0$  8

6. (a) Find the orthogonal trajectory of the family of curves  $x^3y - xy^3 = c$ . 6

- (b) Obtain the Fourier expansion of  $f(x) = \left(\frac{\pi-x}{2}\right)^2$  in the interval 6

$$0 < x < 2\pi, \quad f(x+2\pi) = f(x)$$

Also deduce that  $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

- (c) Solve using Laplace transform  $(D^2 - 3D + 2)y = 4e^{2t}$ , with  $y(0) = -3, y'(0) = 5$ . 8

**Con. 9830-14.**