MCA (SEM-I)
Discrete Mathematics
(OCT-16)

QP CODE : 510300

Duration 3 hours

Total 100 marks

N.B: (1) Question No. 1 is compulsory.
(2) Attempt any four out of remaining six questions.
(3) Assume any necessary data but justify the same.
(4) Figures to the right indicate marks.

1. a) (i) Obtain a disjunctive normal form of [5]
$P\vee(\neg P\rightarrow(Q\vee(Q\rightarrow\neg R)))$

(ii) Let S={1,2,3,4} and A=S×S. Define the following relation R on A.

(a,b)R(a',b') if and only if $\frac{a}{b}=\frac{a'}{b'}$.

Show that R is an equivalence relation. Compute A/R [5]

b) (i) Determine whether the set of real numbers with $a*b=a+b+2$ is a semigroup, a monoid or neither. If it is a monoid, specify the identity. If it is a semigroup or a monoid determine whether it is commutative. [5]

ii) The solution of the recurrence relation $C_0a_n+C_1a_{n-1}+C_2a_{n-2}=f(n)$ is 4^n+5^n+6. Given that f(n)=30 for all n. Determine C_0, C_1, C_2. [5]

2. a) (i) Use laws of logic to show the following equivalence. $(P\rightarrow Q)\wedge(R\rightarrow Q)\equiv(P\vee R)\rightarrow Q$. [5]

ii) What is functionally complete set of connectives. Explain with two examples. [5]

b) Let A= { 1, 2, 3, 5, 6, 10, 15, 30} and R be relation defined by aRb if and only if "a divides b". Show that R is a partial order relation. Draw the Hasse diagram of (A, R). [10]

3. a) (i) Using mathematical induction to prove that 3^n+2n-1 is divisible by 4 for every positive integer n. [5]

ii) Show that the hypothesis "If you send me e-mail message, then I will finish writing the program", "If you do not send me an e-mail message, then I will go to sleep early" and "If I go to sleep early, then I will wake up feeling refreshed", leads to the conclusion "If I do not finish writing the program then I will wake up feeling refreshed". [5]

b) (i) Let the universe of discourse be D={0, 1, 2, …,9}. Let Q(x,y) be the statement "x+y=x-y". Determine the truth values of the following.

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(1) Q(1,1), (2) $\exists y\ \forall x\ Q(x,y)$ (3) $\forall y\ \exists x\ Q(x,y)$,
(4) $\exists x\ Q(x,2)$ (5) $\forall x\ \exists y\ Q(x,y)$ [5]

ii) Find the particular solution of $a_n - 6a_{n-1} + 9a_{n-2} = (n+1)\times 3^n$ [5]

4. a)(i) Use the method of homogeneous solutions to find a particular solution of recurrence relation $3a_n = 5a_{n-1} - 2a_{n-2} + n$ with initial condition $a_0 = -1$, $a_1 = 1$ [5]

ii) Let $a_n = \begin{cases} 0 & 0 \le n \le 2 \\ 2^{-n} + 3 & n \ge 3 \end{cases}$ Find Δa_n, where Δ denotes the forward difference. [5]

b) Obtain the recurrence relation for the maximum number of regions of a plane when there are n lines in the plane. Give suitable initial condition(s). Solve the recurrence relation. [10]

5 a) (i) Let G be a group and let a be a fixed element of G. Show that the function $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$ for $x \in G$ is an isomorphism. [5]

ii) Consider the group G={1,2,3,4,5,6} under multiplication modulo 7. Find the multiplication table of G. Find the order of the subgroup generated by 3. Is G cyclic? [5]

b)(i) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. [5]

Determine the (3,6) group code $e_H : B^3 \to B^6$.

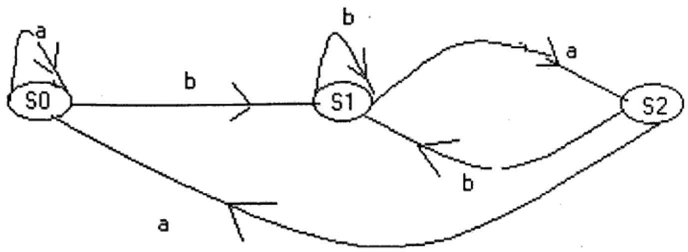
(ii) Consider the group code defined by e: $B^2 \to B^5$ such that e(00)=00000, e(01)=01110, e(10)=10101, e(11)=11011. Decode the following word 10011 relative to maximum likelihood decoding function. [5]

6. a) (i) Let the (2,9) encoding function e defined by [5]

e(00) = 000 000 000 e(01) = 011 101 100
e(10) = 101 110 001 e(11) = 110 001 111

be an associated maximum likelihood function. How many errors will (e,d) correct.

(ii) Construct the transition table of the finite state machine whose diagraph is shown. [5]



b) (i) Let V={v_0, w, a, b, c} S={a, b, c} and let ↦ be the relation on V* given by

1. v_0 ↦ aw   2. w ↦ bbw   3. w ↦ c   [5]

Consider the phase structure grammar G=(V, S, v_0, ↦).
Derive the sentence ab^4c. Also draw the derivation tree.

(ii) Let the state transition table for a finite state machine be   [5]

	0	1
S_0	S_0	S_1
S_1	S_1	S_2
S_2	S_2	S_3
S_3	S_3	S_0

List values of the transition function f_w for (a) w=01001, (b) w=11100.

7. a) Determine whether the relation R on a set A is reflective, irreflective, symmetric, asymmetric, antisymmetric or transitive. Give necessary explanation to your answer.
A= set of all positive integers, aRb iff GCD (a,b)=1   [10]

b) Perform the following.   [10]

i) $(1011.110)_2 = (?)_{10}$

ii) $(413)_8 = (?)_{10}$

iii) $(1101)_2 - (1001)_2 . = (?)_2$

iv) $(1011)_2 \times (1010)_2 = ?$

v) $(10100)_2 \div (100)_2 = ?$

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