

S.E. - IV Sem - Biotech

19/5/16

Applied Maths - IV

24

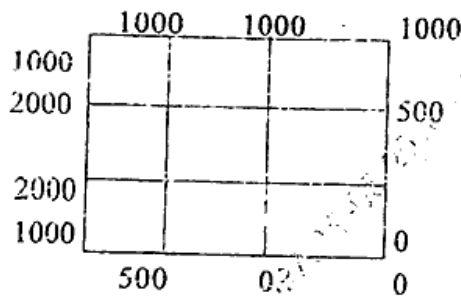
SE/IV/CBGS/BT/AM-IV
QP Code : 568200

3 Hours)

[Total Marks : 80

- N.B. : (1) Question No. 1 is compulsory.
(2) Answer any three questions from the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find the Fourier Transform of $f(x) = e^{-|x|}$ 5
(b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2xz\mathbf{i} + (xz - y)\mathbf{j} + 2zk\mathbf{k}$ from $O(0,0,0)$ to $P(3, 1, 2)$ along the line OP. muADDA.com 5
(c) Obtain half range sine series in $(0, \pi)$ for $f(x) = x(\pi - x)$ 5
(d) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for the following data by successive iterations. 5
(upto 2 iterations).



2. (a) Evaluate by Green's Theorem $\int_C \vec{F} \cdot d\vec{r}$ where $F = -xy(\mathbf{x}\mathbf{i} - y\mathbf{j})$ and C is $r = a(1 + \cos \theta)$ 6
(b) Obtain complex form of Fourier series for $f(x) = \cosh 2x + \sin h2x$ in $(-2, 2)$. 6
(c) A rod of length 'l' has its ends A and B kept at 0°C and 100°C respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so while that of A is maintained. Find the temperature $u(x,t)$ at a distance x from A and at time 't'. muADDA.com 8
3. (a) Show that the set of functions 6
 $1, \sin \frac{\pi x}{L}, \cos \frac{\pi x}{L}, \sin \frac{2\pi x}{L}, \cos \frac{2\pi x}{L}, \dots$
 form an orthogonal set in $(-L, L)$ and construct an orthonormal set.

[TURN OVER

19/05/2010

SE/IV/CBGS/BT/Am-11
QP Code : 568200

- 2 -

- (b) Express the function $f(x) = \begin{cases} \sin x & ; |x| < \pi \\ 0 & ; |x| > \pi \end{cases}$ 6

as Fourier sine integral and evaluate

$$\int_0^x \frac{\sin \omega x \sin \pi \omega}{1 - \omega^2} d\omega$$

- (c) Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = 4xi - 2y^2j + z^2k$ and S is the region bounded 8
by $y^2 = 4x$, $x = 1$, $z = 0$, $z = 3$ using Gauss's Divergence Theorem.

4. (a) A tightly stretched string with fixed end point $x = 0$ and $x = \ell$, in the shape 6
defined by $y = Kx(\ell - x)$ where K is a constant is released from this position

of rest. Find $y(x, t)$, the vertical displacement if $\frac{\partial^2 y}{\partial x^2} = \frac{c^2}{\partial t^2}$

- (b) Find the Fourier expansion of $f(x) = 2x - x^2$; $0 \leq x \leq 3$. 6

- (c) If the vector field \vec{F} is irrotational find the constants a, b, c where \vec{F} is given 8
by $\vec{F} = (x + 2y + az) i + (bx - 3y - 2z) j + (4x + cy + 2z) k$. Find the Scalar
potential for \vec{F} , and work done in moving a particle in this field from
(1, 2, -4) to (3, 3, 2) along the straight line joining these points.

5. (a) A long rectangular plate of width 'a' cms with insulated surface has its 6
temperature 'u' equal to zero on both the long sides and one short side so
that $u(0, y) = 0$, $u(a, y) = 0$, $u(x, \infty) = 0$ and $u(x, 0) = kx$. Find the temperature
 $u(x, y)$ at any point of the plate in the steady-state.

- (b) Find the Fourier cosine transform of 6
 $f(x) = e^{-x} + e^{-2x}$; $x > 0$

[TURN OVER

SE/IV/CEG S/ BT/AM-IV
QP Code : 568200

- (c) By using Stoke's Theorem, evaluate

$$\iint_S [(x^2 + y^2)\mathbf{i} + (x^2 + y^2)\mathbf{j}] \cdot d\mathbf{r}$$

Where C is boundary of the region enclosed by circles $x^2 + y^2 = 4$,
 $x^2 + y^2 = 16$

8

6. (a) Evaluate $\int_A^B (3x^2y - 2xy)dx + (x^3 - x^2)dy$ along $y^2 = 2x^3$ from A(0,0) and B(2,4).

6

- (b) Obtain Fourier series for

$$f(x) = x + \frac{\pi}{2}, -\pi < x < 0$$

$$\frac{\pi}{2} - x; 0 < x < \pi$$

Hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

- (c) Expand $f(x) = x \sin x$ in the interval $0 \leq x \leq 2\pi$

8

Deduce that $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \frac{3}{4}$