(1) S.E - IV Sem - Biotech.

Applied maths IV - c/15/cbgs/BT/Am-IV

(28) Q.P. Code : 5380

30/11/2015

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. 1 is compulsory.
(2) Attempt any three questions from the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find Fourier sine transform of $\frac{e^{-ax}}{x}$ 5

(b) Evaluate $\int_c \bar{F}\cdot d\bar{r}$ where $\bar{F} = \cos y\, i - x \sin y\, j$ and C is the curve $y=\sqrt{1-x^2}$ in xy-plane from (1, 0) to (0, 1) 5

(c) Expand f (x) = x , 0 < x < 2 in the half range cosine series 5

(d) Classify the following differential equation. $U_{xx}-2U_{xy}+U_{yy}+3U_y+4u_x=3x-2y$ 5

2. (a) Using Green's theorem, evaluate $\int_c \bar{F}\cdot d\bar{r}$ where C is the curve enclosing the region bounded by $y^2=4ax$, $x=a$ in the plane z=0 and 6

$\bar{F}=(2x^2y+3z^2)i+(x^2+4yz)j+(2y^2+6x^2)k$

(b) Find the complex form of fourier series for $f(x)=e^{ax}$ in $(-\pi,\pi)$ 6

(c) Solve the equation $\frac{\partial u}{\partial t}=\frac{k\,\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod of length 'ℓ' subject to following conditions. 8

(i) u is not infinity for $t \to \infty$

(ii) $\frac{\partial u}{\partial x}=0$ for x = 0 and x = ℓ for any time t

(iii) $u=\ell x - x^2$ for t = 0 between x = 0 and x = ℓ

3. (a) Show that the set of functions $\sin\frac{\pi x}{L}, \sin\frac{3\pi x}{L}, \sin\frac{5\pi x}{L}, \ldots$, is orthogonal over (0, L) 6

(b) Express the function $f(x) = \pi/2$; $0 < x < \pi$ 6

0 ; $x > \pi$

as fourier sine Integral and show that $\int_0^\infty \frac{1-\cos\pi\omega}{\omega}\sin\omega x\, d\omega = \frac{\pi}{2}; 0<x<\pi$

(c) Using Gauss Divergence Theorem, prove that 8

$\iint_S (y^2z^2 i + z^2x^2 j + z^2y^2 k)\cdot \bar{N} ds = \frac{\pi}{12}$ where S is the part of the sphere $x^2+y^2=1$ in xy-plane

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4. (a) A string is stretched and fastened to the two points distance 'ℓ' apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time t =0. Show that the displacement of a point at a distance x from one end at time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi t}{\ell}\right)$ 6

(b) Find the Fourier expansion of $f(x) = 4 - x^2$ in (0,2) 6

(c) Prove that $\overline{F} = (y^2\cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k$ is conservative field. Find (i) Scalar potential for $\overline{F}$(ü) the work done is moving an object in this field from (0,1,-1) to (π/2, -1, 2) 8

5. (a) A retangular metal plate with insulated surfaces is of width 'a' and so long as compared to its breadth that it can be considered infinite in length without introducing an applicable error. If the temperature along one short edge y = 0 is given by $u(x,0) = u_0 \sin(\pi x/a)$ for $0 < x < a$ and other long edges x = 0 and x = a and the short edges are kept at zero degrees temperature find the function u (x, y) describing the steady state. 6

(b) Find the Fourier cosine transform of $f(x) = e^{-x} + e^{-2x}$; $x > 0$ 6

(c) Verify Stoke's Theorem for vector field $\overline{F} = 4xz\,i - y^2 j + yzk$ over the area in the plane z = 0 bounded by x = 0 , y = 0, $x^2+y^2=1$. 8

6. (a) A vector field is given by $\overline{F} = \sin y\, i + x(1+\cos y)j$, evaluate the line integral over the circular path $x^2+y^2=a^2$, z = 0 6

(b) Evaluate $\iint_S (\nabla \times \overline{F})\cdot d\overline{S}$ where $\overline{F} = (2x - y + z)i + (x + y - z)j + (3x - 2y + 4z)k$ and S is the surface of the cylinder $x^2+y^2=4$ bounded by the plane z = 9 and open at other end. 6

(c) Find Fourier expansion of 8

$f(x) = 1 - x^2$, $|x| < 1$

$\quad\quad 0 \quad |x| > 1$

Hence, evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\cdot dx$$

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