

SE - sem-III (old) - Electrical Engg Marks - III

R-2007

Q.P. Code : 542402

25/11/16

(3 Hours)

[Total Marks : 100]

- N.B. 1 Question No. 1 is compulsory. Attempt any FOUR questions from Question No 2 to Question No 7.
- 2 Figures to the right indicate full marks.

1. a) Find  $L\{t + e^{-3t} + \sin 2t\}$  (5)
- b) Show that the function  $u = \frac{1}{2} \log(x^2 + y^2)$  is a harmonic function (5)
- c) Find Fourier series of the function  $f(x) = x; -\pi < x < \pi$  (5)
- d) Evaluate complex line Integral  $\int_0^{1+i} (x - y + ix^2) dz$  along the straight line from  $z=0$  to  $z=1+i$  (5)
2. a) Evaluate  $\oint_C \frac{z}{(z-1)(z-2)} dz$  where  $C$  is  $|z| = 3$  (6)
- b) Find the Fourier series of  $f(x) = 2x - x^2$  in  $[0, 2]$ . (6)
- c) Using Laplace transform, show that  $\int_0^{\infty} e^{-2t} \sin 3t dt = \frac{3}{13}$  (8)
3. a) Using Counter Integration and residue theorem evaluate  $\int_0^{2\pi} \frac{1}{5 + 4 \cos \theta} d\theta$  (6)
- b) Find the half range Fourier cosine series for  $f(x) = \sin x; 0 \leq x \leq \pi$  (6)
- c) Find  $L^{-1} \left\{ \frac{s^2}{(s^2 + a^2)^2} \right\}$  by using Convolution theorem (8)
4. a) Using Laplace Transform solve the differential equation  $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 5y = e^{-t}; y(0) = 0, y'(0) = 0$  (6)
- b) Show that  $f(z) = (x^3 - 3xy^2 + 2xy) + i(3x^2y - x^2 + y^2 - y^3)$  is analytic and hence find  $f'(z)$  (6)
- c) Find the Fourier integral representation of  $f(x) = \begin{cases} e^{ax} & ; x \leq 0 \\ e^{-ax} & ; x \geq 0 \end{cases}$  (8)
- and hence show that  $\int_0^{\infty} \frac{\cos sx}{s^2 + a^2} ds = \frac{\pi}{2a} e^{-ax}$  for  $x > 0$

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TURN OVER

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5. a) Find  $L\left\{\frac{e^{-2t} - e^{-3t}}{t}\right\}$  (6)

b) Find a bilinear transformation which maps  $z=1, i, -1$  into  $w=1, 0, -i$  (6)

c) Find Laurent series for  $f(z) = \frac{1}{z^2 - 3z + 2}$  in  $1 < |z| < 2$  (8)

6. a) State and prove first shifting property of Laplace transform. (6)

b)  $w = z^2 + z$  maps the circle  $|z|=1$  in the  $z$ -plane into the cardioid  $\rho = 2(1 + \cos\phi)$  in the  $w$ -plane (6)

c) If  $L\{F(t)\} = \frac{1}{s\sqrt{s+1}}$  find  $L\{tF(2\sqrt{t})\}$  (8)

7. a) Find  $L^{-1}\left\{\frac{s+1}{s^2 - 2s + 2}\right\}$  (6)

b) Find the constants  $a$  &  $b$  if  $1, x, -1 + ax + bx^2$  are orthogonal over  $(-1, 1)$  (6)

c) Find the analytic function  $f(z)=u+iv$ , in terms of  $z$ , if  $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$  (8)

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