

Sem - III / EXTC 8 INST / APP. Maths - III / CGS

18-05-16

QP Code : 30598

(Revised course)

Time : 3 hours

Total marks : 80

N.B : (1) Question No.1 is compulsory.

(2) Answer any three questions from remaining.

(3) Assume suitable data if necessary.

Evaluate

1. (a) $\int_0^{\pi} e^{-2t} \left(\frac{\sinh t \sin t}{t} \right) dt$ 05

(b) Obtain the Fourier Series expression for 05
 $f(x) = 9 - x^2$ in $(-3, 3)$

(c) Find the value of 'p' such that the function $f(z)$ expressed in 05
 polar co-ordinates as
 $f(z) = r^3 \cos p\theta + ir^p \sin 3\theta$ is analytic.

(d) If $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$. 05
 Show that \bar{F} is irrotational and solenoidal.

2. (a) Solve the differential equation using Laplace Transform 06

$$\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 8y = 1, \text{ given } y(0) = 0 \text{ and } y'(0) = 1$$

(b) Prove that 06

$$J_4(x) = \left(\frac{48}{x^5} - \frac{8}{x} \right) J_1(x) - \left(\frac{24}{x^2} - 1 \right) J_0(x)$$

(c) i) Find the directional derivative of 08
 $\vec{v} = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction of $2\hat{i} + 3\hat{j} + 6\hat{k}$.

ii) If $\vec{r} = xi + yj + zk$

$$\text{Prove that } \nabla \log r = \frac{\vec{r}}{r^2}$$

3. (a) Show that $\{\cos x, \cos 2x, \cos 3x, \dots\}$ is a set of orthogonal functions over $(-\pi, \pi)$. Hence construct an orthonormal set. 06

- (b) Find an analytic function $f(z) = u + iv$ where. 06

$$u = \frac{x}{2} \log(x^2 + y^2) - y \tan^{-1}\left(\frac{y}{x}\right) + \sin x \cosh y$$

- (c) Find Laplace transform of 08

i) $\int_0^\infty ue^{-3u} \cos^2 2u du$

ii) $t \sqrt{1 + \sin t}$

4. (a) Find the Fourier Series for 06

$$f(x) = \frac{3x^2 - 6\pi x + 2\pi^2}{12} \quad \text{in } (0, 2\pi)$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$

- (b) Prove that 06

$$\int_0^b x J_0(ax) dx = \frac{b}{a} J_1(ab)$$

- c) Find 08

i) $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$

ii) $L^{-1}\left[\left(\frac{s+2}{s^2-2s+17}\right)\right]$

5. (a) Obtain the half range cosine series for

06

$$\begin{aligned}f(x) &= x, 0 < x < \frac{\pi}{2} \\&= \pi - x, \frac{\pi}{2} < x < \pi\end{aligned}$$

(b) Find the Bi-linear Transformation which maps the points
1, i, -1 of z plane onto i, 0, -i of w-plane

06

(c) Verify Green's Theorem for $\int_C \bar{F} \cdot d\bar{r}$ where

08

$\bar{F} = (x^2 - xy)\hat{i} + (x^2 - y^2)\hat{j}$ and C is the curve bounded by $x^2 = 2y$
and $x = y$

6.(a) Show that the transformation

06

$w = \frac{i - iz}{1 + z}$ maps the unit circle $|z| = 1$ into real axis of w plane.

(b) Using Convolution theorem find

06

$$L^{-1}\left[\frac{s}{(s^2 + 1)(s^2 + 4)} \right]$$

(c)

08

i) Use Gauss Divergence Theorem to evaluate
 $\iint_S \bar{F} \cdot \hat{n} ds$ where $\bar{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S is the sphere
 $x^2 + y^2 + z^2 = 9$ and \hat{n} is the outward normal to S

ii) Use Stoke's Theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ where
 $\bar{F} = x^2\hat{i} - xy\hat{j}$ and C is the square in the plane $z=0$ and
 bounded by $x=0, y=0, x=a$ and $y=a$.