

(3 Hours)

Marks 80

www.muadda.com

N.B.1) Question no 1 is compulsory.

2) Figures to the right indicate full marks.

3) Attempt any three from Q2 to Q6.

Q1 a) If any 14 integers from 1 to 26 are chosen then show that at least one of them is a multiple of another. 05

b) Functions f and g are defined as follows : 05

$$f: \mathbb{R} \rightarrow \mathbb{R}, g: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = 2x + 3, g(x) = 3x - 4.$$

Find $f \circ g$ and $g \circ f$. www.muadda.com

c) $L\left\{\frac{d}{dt} \frac{\sin 3t}{t}\right\}$. 05

d) Show that there does not exist an analytic function whose real part is $3x^2 - 2x^2y + y^2$. 05

Q2 a) Evaluate $\int_0^{\infty} e^{-t} \left(\frac{\cos 3t - \cos 2t}{t}\right) dt$ 06

b) Evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\right\}$ 06

c) Find bilinear transformation which maps the points $Z=1, i, -1$ into points 08

$W = i, 0, -i$. Hence find fixed pts of transformation and the image of $|z| < 1$.

www.muadda.com

Q3 a) If A, B, C are subsets of universal set U , then prove that 06

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

b) Let $A = \{1, 2, 3, 6\}$, $B = \{1, 2, 3, 6, 7, 14, 21, 42\}$ and R be the relation 'is divisible by'. 06

Draw Hasse Diagram for two sets. Show that are posets.

c) Find Laplace transform of following functions. 08

$$(i) e^{-2t} \sqrt{1 - \sin t} \quad (ii) t e^{-2t} H(t - 1)$$

www.muadda.com

Q4 a) In how many different ways can 4 ladies and 6 gentlemen be seated in a row, so no ladies sit together. **06**

b) Find analytic function whose real part is **06**

$$\frac{\sin 2x}{\cosh 2y + \cos 2x}$$

c) Evaluate inverse Laplace Transform of following functions. **08**

(i) $\frac{1}{(s-3)(s+4)^2}$ by convolution theorem (ii) $\log\left(1 + \frac{a^2}{s^2}\right)$

www.muadda.com

Q5 a) Solve the following equation by using Laplace transform **06**

$$\frac{dy}{dt} + 2y + \int_0^t y dt = \sin t, \text{ given that } y(0) = 1$$

b) Find p such that the function $\frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{y}{x}$ is analytic. **06**

c) For $x, y \in \mathbb{Z}$, xRy if and only if $2x + 5y$ is divisible by 7 **08**

is R an equivalence relation? Find equivalence relation.

www.muadda.com

Q6 a) Each coefficient of the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have real roots. **06**

b) A certain test for particular cancer is known to be 95% accurate. A person submits to the test and result is positive. Suppose that a person comes from a population of the 1,00,000 where 2000 people suffer from disease. What can we conclude about the probability that person under test has particular cancer? **06**

c) i) If five points are taken in a square of side 2 units. Show that at least two of **04**

them are no more than $\sqrt{2}$ units apart.

ii) How many friends must you have to guarantee that at least five of them have **04**

their birthday in same month.

www.muadda.com