

S.E. (SEM. III) (CBSGS) MECHANICAL ENGG.  
APPLIED MATHEMATICS III

14th May, 2015  
3.00 pm to 6.00 pm

Mechanical/Automobile

Q.P. Code : 4755

(3 Hours)

[ Total Marks : 80

N.B. : (1) Question No. 1 is compulsory.

(2) Attempt any three questions out of remaining five questions.

- 1 (a) Find the Laplace transform of  $te^{-t} \cosh 2t$  5
- (b) Find the fixed points of  $w = \frac{3z-4}{z-1}$ . Also express it in the normal form. 5

$\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + \lambda$  where  $\lambda$  is a constant and  $\alpha$  is the fixed point. Is this transformation parabolic?

- (c) Evaluate  $\int_0^{1+i} (x^2-iy)dz$  along the path i)  $y=x$ , ii)  $y=x^2$  5
- (d) Prove that  $f_1(x)=1$ ,  $f_2(x)=x$ ,  $f_3(x) = \frac{3x^2-1}{2}$  are orthogonal over  $(-1,1)$  5

2. (a) Find inverse Laplace transform of  $\frac{2s}{s^4+4}$  6
- (b) Find the image of the triangular region whose vertices are  $i$ ,  $1+i$ ,  $1-i$  under the transformation  $w = z+4-2i$ . Draw the sketch. 6
- (c) Obtain fourier expansion of  $f(x) = |\cos x|$  in  $(-\pi, \pi)$ . 8

3. (a) Obtain complex form of fourier series for  $f(x)=\cosh 2x + \sinh 2x$  in  $(-2,2)$ . 6
- (b) Using Crank-Nicholson simplified formula solve  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$  given 6

$u(0,t)=0$ ,  $u(4,t)=0$ ,  $u(x,0) = \frac{x}{3} (16-x^2)$  find  $u_{ij}$  for  $i=0,1,2,3,4$  and  $j=0,1,2$

- (c) Solve the equation  $y + \int_0^t y dt = 1 - e^{-t}$  8

## Q.P. Code : 4755

2

4. (a) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$  6
- (b) Find half - range cosine series for  $f(x)=e^x$ ,  $0<x<1$  6
- (c) Obtain two distinct Laurent's series for  $f(z) = \frac{2z-3}{z^2-4z-3}$  in powers of  $(z-4)$  indicating the regions of convergence. 8
5. (a) Solve  $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$  by Bender - Schmidt method, given  $u(0, t) = 0$ ,  $u(4, t) = 0$ ,  $u(x, 0) = x(4-x)$ . Assume  $h=1$  and find the values of  $u$  upto  $t = 5$  6
- (b) Find the Laplace transform of  $e^{-4t} \int_0^1 u \sin 3u du$  6
- (c) Evaluate  $\int_C \frac{z+3}{z^2+2z+5} dz$  where  $C$  is the circle i)  $|z| = 1$ , ii)  $|z+1-i|=2$  8
6. (a) Find inverse Laplace transform of  $\frac{s}{(s^2-a^2)^2}$  by using convolution theorem. 6
- (b) Find an analytic function  $f(z) = u+iv$  where  $u+v=e^x(\cos y + \sin y)$  6
- (c) Solve the equation  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$  for the conduction of heat along a rod of length  $l$  subject to following conditions 8
- (i)  $u$  is not infinity for  $t \rightarrow \infty$
- (ii)  $\frac{\partial u}{\partial x} = 0$  for  $x=0$  and  $x=l$  for any time  $t$
- (iii)  $u=lx-x^2$  for  $t=0$  between  $x = 0$  and  $x=l$