S.E (MECHANICAL ENGG) (SEM III) (CBSGS)   25ᵗʰNovember, 2013

11-11-13-DTP7-RM-24   Applied Mathematics III   3.00 pm to 6.00 pm

Con. 7845-13.   Mechanical/Automobile   GX-12025

(3 Hours)   [Total Marks : 80

N.B. : (1) Question No. 1 is **Compulsory.**

(2) **Answer** any **Three** from remaining.

(3) **Figures** to the **right** indicate **marks.**

1. (a) Find laplace of $\sin \sqrt{t}$ 5

(b) Show that the set of functions $\sin\left(\dfrac{\pi x}{2L}\right)$, $\sin\left(\dfrac{3\pi x}{2L}\right)$, $\sin\left(\dfrac{5\pi x}{2L}\right)$ is orthogonal over   5

(O, L).

(c) Show that $u = \sin x \cos hy + 2 \cos x \sin hy + x^2 - y^2 + 4xy$ Satisfies laplace equation   5
and find its corresponding analytic function $f(z) = u + iv$.

(d) Determine constants a,b,c,d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic.   5

2. (a) Find complex form of fourier series $f(x) = e^{3x}$ in $0 < x < 3$.   6

(b) Using Crank Nicholson Method solve $u_t = u_{xx}$ subject to $u(x,0) = 0$ $u(0, t)=0$ and   6
$u(l,t) = t$ for two time steps.

(c) Solve using laplace transforms $\dfrac{d^2 y}{dt^2} + y = t$, $y(0) = 1$, $y^1(0) = 0$ 8

3. (a) Find bilinear transformation that maps the points $0, 1-\infty$ of the z plane into $-5, -1,$ 6
3 of w plane.

(b) By using Convolution Theorem find inverse laplace transform of $\dfrac{1}{(S^2 + 4S + 13)^2}$ 6

(c) Find fourier series of $f(x) = x^2$ $-\pi \le x \le \pi$ and prove that   8

(i) $\dfrac{\pi^2}{6} = \sum\limits_{1}^{\infty} \dfrac{1}{n^2}$

(ii) $\dfrac{\pi^2}{12} = \sum\limits_{1}^{\infty} \dfrac{(-1)^{n+1}}{n^2}$

(iii) $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots$

4. (a) Evaluate $\int\limits_{0}^{\infty} e^{-t} \dfrac{\sin^2 t}{t} dt$ 6

(b) Solve $\dfrac{\partial^2 u}{\partial x^2} - 32 \dfrac{\partial u}{\partial t} = 0$ by   6

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Bender schmidt method subject to conditions u(o, t) = 0 u (x, 0) = 0
u(l, t) = t taking h = 0.25 0 < x < 1

(c) Obtain two distinct Laurent's Series for $f(z) = \dfrac{2z-3}{Z^2-4z-3}$ in Powers of (z-4)
indicating Region of Convergence.   8

5. (a) Evaluate $\displaystyle\int_0^{1+i} Z^2 dZ$ along   6

    (i) line y = x
    (ii) Parabola x = y^2
    Is line independent of path? Explian.

   (b) Find half range Cosine Series for f(x) = e^x 0 < x < 1.   6

   (c) Find analytic function   8
     f(z) = u + iv such that

$$u-v = \frac{Cosx + Sinx - e^{-y}}{2Cosx - e^{y} - e^{-y}}$$

     when $f\left(\frac{\pi}{2}\right) = 0$

6. (a) A tightly streched sting with fixed end points x = 0 and x = ℓ in the shape defined   6
     by y = K x (l - x) where K is a Constant is released from this position of rest. Find y(x,t)
     The vertical displacement

     if $\dfrac{\partial^2 y}{\partial t^2} = C^2 \dfrac{\partial^2 y}{\partial x^2}$

   (b) Find image of region bounded by x = 0, x= 2 y = 0 y = 2 in the z plane under the   6
     transformation w = (1 + i) Z

   (c) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{25-16cos^2\theta}$ 8