

(3 Hours)

[Total Marks : 80

- N.B :** (1) Question no.1 is compulsory.
(2) Answer any three from remaining.
(3) Figures to the right indicate marks.

1. (a) Find laplace transform of $t^3 \cos t$. 5
(b) Find the image of $|z - ai| = a$ under the transformation $w = \frac{1}{z}$. 5
(c) Construct an analytic function whose real part is $e^{2x} (x \cos 2y - y \sin 2y)$. 5
(d) Show that the set of functions $\cos nx$ $n = 1, 2, 3 \dots$ is orthogonal on $(0, 2\pi)$. 5

2. (a) By using Convolution Theorem. Find invese laplace transform of $\frac{1}{s^2(s+1)^2}$. 6
(b) Find bilinear transformation that maps the points $2, i, -2$ onto the point $1, i, -1$. 6
(c) Find Fourier Series for $f(x) = \cos mx$ in $(\pi, -\pi)$ where m is not an integer. Deduce 8

that $\cos m\pi = \frac{2m}{\pi} \left(\frac{1}{2m^2} + \frac{1}{m^2-1^2} + \frac{1}{m^2-2^2} + \dots + \frac{1}{m^2-n^2} \right)$ hence show that

$$\sum_{n=1}^{\infty} \frac{1}{9n^2-1} = \frac{1}{2} - \frac{\pi\sqrt{3}}{18}$$

3. (a) Find Complex form of fourier series $f(x) = e^{3x}$ in $0 < x < 3$. 6
(b) Using Crank Nicholuson method solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to $0 \leq x \leq 1$ $u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = 10^3 x(1-x)$ taking $h = 0.25$ in one step. 6
(c) Using laplace transform solve $(D^2+2D+5)y = e^{-t} \sin t$ when $y(0) = 0$ and $y'(0) = 1$. 8

4. (a) Evaluate $\int f(z) dz$ along the Parabola $y = 2x^2$ from $z = 0$ to $z = 3 + 18i$ where 6
 $f(z) = x^2 - 2iy$
(b) Find half range cosinc series for 6

$$f(x) = x \quad 0 < x < \frac{\pi}{2}$$

$$= \pi - x \quad \frac{\pi}{2} < x < \pi$$

- (c) Obtain two distinct Laurent's series of $f(z) = \frac{1}{(1+z^2)(z+2)}$ for $1 < |z| < 2$ and $|z| > 2$. 8

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Time : 4535

5. (a) By using Bender Schmidt method solve $\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial t}$ $f(0, t) = f(5, t) = 0$. 6
 $f(x, 0) = x^2(25 - x^2)$ find f in range taking $h = 1$ and upto 5 seconds.

(b) Evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$. 6

(c) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta$. muADDA.com 8

6. (a) A string is stretched and fastened to two points distance ℓ apart. motion is started by displacing the string in the form $y = a \sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time $t = 0$. Show that the displacement of a point at a distance x from one end at a distance x from one end at time t is given by $y(x, t) = a \sin\left(\frac{\pi x}{\ell}\right) \cos\left(\pi \frac{ct}{\ell}\right)$. 6

- (b) If $f(z) = u + iv$ is analytic and $u - v = e^x (\cos y - \sin y)$ find $f(z)$ in terms of z . 6
 (c) Evaluate : 8

$$L^{-1}(2 \tanh^{-1} s)$$

$$L^{-1}\left[\frac{s}{(s-2)^6}\right] \quad \text{muADDA.com}$$

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