

Sem III / MECH & PROD - CBGS / 12-05

Applied Maths - III

QP Code : 30542

(3 Hours) [Total Marks : 80

N.B. : (1) Question No.1 os compulsory
(2) Answer any three from remaining
(3) Figures to the right indicate marks.

1. (a) Find laplace transform of $\frac{\sin^2 2t}{t}$ 5

(b) Find the orthogonal tragectory of the family of curves 5
$e^{-x}\cos y + xy = \alpha$ where α is a real constant in the xy plane.

(c) Find complex form of fourier series 5
$f(x) = e^{3x}$ in $0 < x < 3$

(d) Show that the function is analytic and find their derivative $f(z) = ze^z$ 5

2. (a) Using laplace transform solve: $\frac{d^2y}{dt^2} + y = t \quad y(0) = 1 \quad y'(0) = 0$ 6

(b) Using Crank Nicholson method 6

Solve : $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$

$u(0, t) = 0 \quad u(4, t) = 0$

$u(x,0) = \frac{x}{3}(16 - x^2)$ find u_{ij}

for i = 0, 1, 2, 3, 4 and j = 0, 1, 2

(c) Show that the set of functions $1, \sin\frac{\pi x}{L}, \cos\frac{\pi x}{L}, \sin\frac{2\pi x}{L}, \cos\frac{2\pi x}{L}, \ldots$ 8
form an orthogonal set in (-L, L) and construct an orthonormal set.

TURN OVER

3. (a) Find the bilinear transformation that maps points 0, 1, ∞ of the z plane into -5, -1, 3 of w plane. 6

(b) By using Convolution theorem find inverse laplace transform of 6

$$\frac{1}{(s-2)^4(s-3)}$$

(c) Find the Fourier series of f(x) 8

$f(x) = \cos x \quad -\pi < x < 0$

$\quad\quad\;\; \sin x \quad 0 < x < \pi$

4. (a) Find half range sine series for x sin x in $(0, \pi)$ and hence deduce 6

$$\frac{\pi^2}{8\sqrt{2}} = \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} \ldots$$

(b) Evaluate and prove that 6

$$\int_0^\infty e^{-\sqrt{2}t} \frac{\sin t \sinh t}{t} = \frac{\pi}{8}$$

(c) Obtain Laurent's series for the function f(z) = 8

$\frac{-7z-2}{z(z-2)(z+1)}$ about z = -1

5. (a) Solve : $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ subject to the conditions u(0, t) = 0, u (5, t) = 0 u (x,0) = $x^2(25 - x^2)$ taking h = 1 upto 3 seconds only by Bender schmidt formula. 6

(b) Construct an analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ 6

(c) Evaluate $\int_0^{\pi} \frac{d\theta}{3 + 2\cos\theta}$ 8

6. (a) An elastic string is stretched between two points at a distance l apart. In its equilibrium position a point at a distance $a (a < l)$ from one end is displaced through a distance b transversely and then released from this position. Obtain $y(x, t)$ the vertical displacement if y satisfies the equation. 6

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

(b) Evaluate : $\int_0^{1+i} z^2 \, dz$ along 6

(i) The line $y = x$

(ii) The parabola $x = y^2$

Is the line integral independent of path? Explain.

(c) Find fourier expansion of 8

$$f(x) = \left(\frac{\pi - x}{2}\right)^2$$

in the interval $0 \leq x \leq 2\pi$ and $f(x+2\pi) = f(x)$

and also deduce

(i) $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots$

(ii) $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \ldots\ldots\ldots$
