

# MUMBAI UNIVERSITY

SEMESTER-1

ENGINEERING MECHANICS SOLVED PAPER-MAY 2017

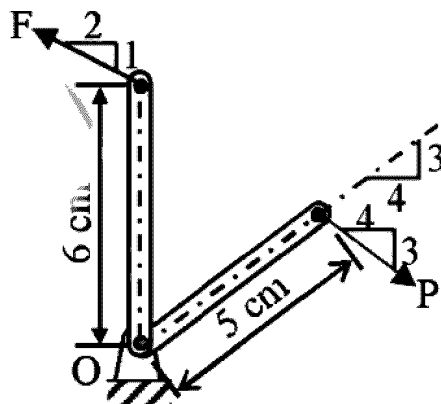
**N.B:-(1) Question no.1 is compulsory.**

**(2) Attempt any 3 questions from remaining five questions.**

**(3) Assume suitable data if necessary, and mention the same clearly.**

**(4) Take  $g=9.81 \text{ m/s}^2$ , unless otherwise specified.**

**Q.1(a) In the rocket arm shown in the figure the moment of 'F' about 'O' balances that  $P=250 \text{ N}$ . Find F. (4 marks)**

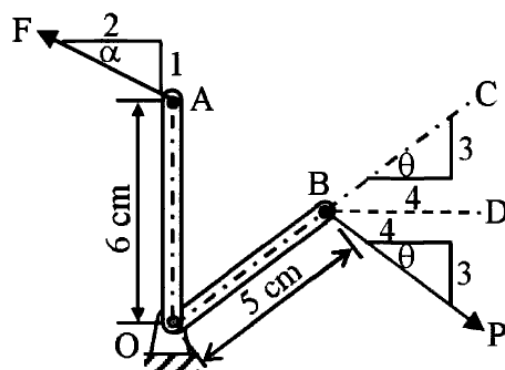


**Solution :**

**Given :**  $P = 250 \text{ N}$

**To find :** Magnitude of force  $F$

**Solution :**



$$\tan \alpha = \frac{1}{2}$$

$$= 0.5$$

$$\alpha = 26.5651^\circ$$

$$\tan \theta = \frac{DE}{AD} = \frac{DE}{BC} = \frac{3}{4} = 0.75$$

$$\theta = 36.87^\circ$$

$$\angle CBD = \angle PBD = \theta = 36.87^\circ$$

$$\angle CBP = 2\theta = 2 \times 36.87 = 73.74^\circ$$

**It is given that at O the moment of F about O balances the moment of P**

$$F \cos \alpha \times OA = P \sin 2\theta \times OB$$

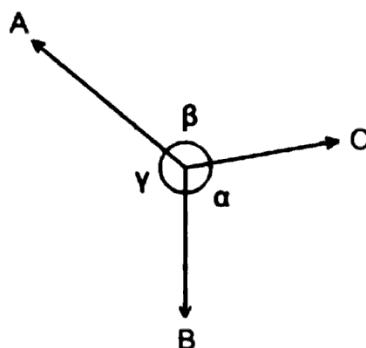
$$F \cos 26.5651 \times 6 = 250 \sin 73.74 \times 5$$

$$F = 223.6068 \text{ N}$$

**Magnitude of force  $F = 223.6068 \text{ N}$**

**Q.1(b) State Lami's theorem.****State the necessary condition for application of Lami's theorem.****(4 marks)****Answer :**

Lami's theorem states that if three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two forces.



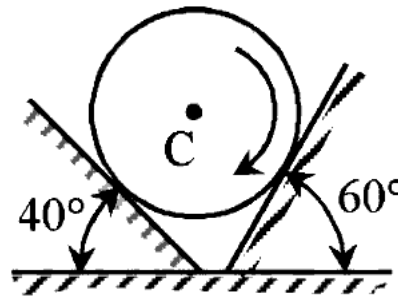
According to Lami's theorem, the particle shall be in equilibrium if :

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

**The conditions of Lami's theorem are:**

- (1) Exact 3 forces must be acting on the body.
- (2) All the forces should be either converging or diverging from the body.

**Q.1(c)** A homogeneous cylinder 3 m diameter and weighing 400 N is resting on two rough inclined surfaces shown. If the angle of friction is  $15^\circ$ . Find couple  $C$  applied to the cylinder that will start it rotating clockwise. (4 marks)



**Solution :**

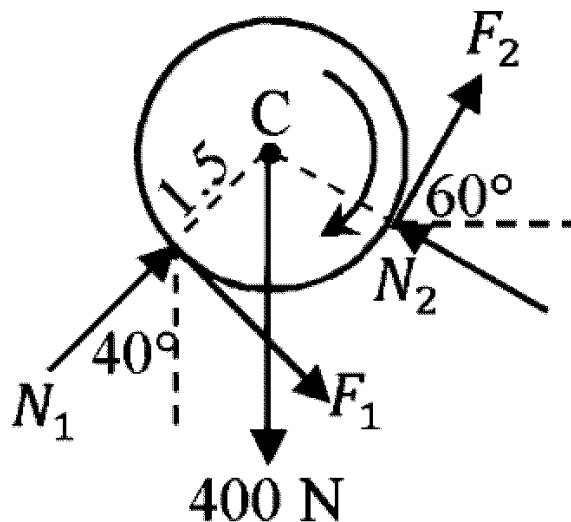
**Given :** Angle of friction is  $15^\circ$

$$\mu = \tan 15 = 0.2679$$

$$\text{Radius} = 1.5 \text{ m}$$

**To find :** Couple  $C$

**Solution:**



$$F_1 = \mu N_1 = 0.2679 N_1 \quad \dots\dots\dots(1)$$

$$F_2 = \mu N_2 = 0.2679 N_2 \quad \dots\dots\dots(2)$$

Assuming the body is in equilibrium

$$\Sigma F_x = 0$$

$$F_1 \cos 40 + N_1 \sin 40 + F_2 \cos 60 - N_2 \sin 60 = 0$$

$$N_1(0.2679 \cos 40 + \sin 40) + N_2(0.2679 \cos 60 - \sin 60) = 0 \quad \dots\dots\dots(3)$$

$$\Sigma F_y = 0$$

$$-F_1 \sin 40 + N_1 \cos 40 + F_2 \sin 60 + N_2 \cos 60 - 400 = 0$$

$$N_1(-0.2679 \sin 40 + \cos 40) + N_2(0.2679 \sin 60 + \cos 60) = 400 \quad \dots\dots\dots(4)$$

Solving (3) and (4)

$$N_1 = 277.4197 \text{ N and } N_2 = 321.3785 \text{ N}$$

Substituting  $N_1$  and  $N_2$  in (1 and 2)

$$F_1 = 0.2679 \times 277.4197 = 74.3344 \text{ N}$$

$$F_2 = 0.2679 \times 321.3785 = 86.1131 \text{ N} \quad \dots\dots\dots(5)$$

$C$  is the couple required to rotate the cylinder clockwise

$$C = F_1 \times r + F_2 \times r$$

$$= 240.6712 \text{ Nm (clockwise)} \quad (r = 1.5 \text{ m}) \text{ (From 5)}$$

**The couple  $C$  required to rotate the cylinder clockwise is 240.6712 Nm (clockwise)**

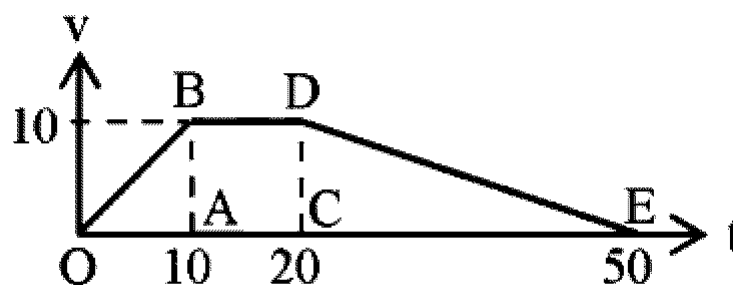
**Q.1(d) From (v-t) diagram find**

**(1) Distance travelled in 10 second.**

**(2) Total distance travelled in 50 second.**

**(3) Retardation**

**(4 marks)**



**Solution:**

We know that the area under v-t graph gives the distance travelled

DISTANCE TRAVELLED IN 0 TO 10 sec = A( $\Delta$ OAB)

$$= \frac{1}{2} \times OA \times AB$$

$$= \frac{1}{2} \times 10 \times 10$$

$$= 50 \text{ m}$$

DISTANCE TRAVELLED IN 0 TO 50 sec = A(Trapezium OBDE)

$$= \frac{1}{2} \times (OE+BD) \times AB$$

$$= \frac{1}{2} \times (50+10) \times 10$$

$$= 300 \text{ m}$$

CONSIDER THE MOTION FROM 20 sec TO 50 sec

We know that slope of v-t graph gives acceleration

E=(50,0) and D=(20,10)

$$\text{Slope of line DE} = \frac{0-10}{50-20} = \frac{-10}{30} = -\frac{1}{3} = -0.3333 \text{ m/s}^2$$

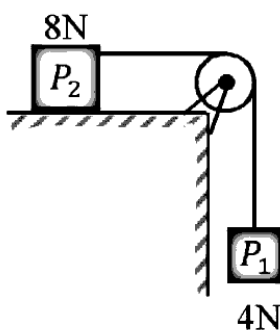
**Distance travelled by object in 10 sec = 50 m**

**Distance travelled by object in 50 sec = 300 m**

**Acceleration = - 0.3333 m/s<sup>2</sup>**

**Q1(e) )Blocks P<sub>1</sub> and P<sub>2</sub> are connected by inextensible string. Find velocity of block P<sub>1</sub>, if it falls by 0.6 m starting from rest.**

**The co-efficient of friction is 0.2. The pulley is frictionless. (4 marks)**



**Solution:**

**Given :** P<sub>1</sub> falls by 0.6 m starting from rest

$$\mu = 0.2$$

**To find :** Velocity of block P<sub>1</sub>

**Solution :**

Consider the motion of block P<sub>2</sub>

Weight of motion P<sub>2</sub> = 8 N

$$\text{Mass of P}_2 = \frac{8}{g}$$

P<sub>2</sub> has no vertical motion

$$\Sigma F_y = 0$$

$$N_2 - 8 = 0$$

$$N_2 = 8 \text{ N}$$

$$F_2 = \mu N_2 \\ = 1.6 \text{ N}$$

Consider the horizontal motion

$$\Sigma F_x = m_2 a$$

$$T - F_2 = m_2 a$$

$$T = 1.6 + \frac{8}{g} a \quad \dots\dots\dots(1)$$

For block P<sub>1</sub>

Weight of P<sub>1</sub> = 4 N

$$\text{Mass of P}_1 = \frac{4}{g} \quad \dots\dots\dots(2)$$

For downward motion

$$\Sigma F_y = m_1 a$$

$$4 - T = m_1 a$$

$$4 - 1.6 - \frac{8}{g} a = \frac{4}{g} a \quad (\text{From 1 and 2})$$

$$a = 1.962 \text{ m/s}^2$$

$$v^2 = u^2 + 2as$$

$$u = 0 \text{ and } s = 1.6 \text{ m}$$

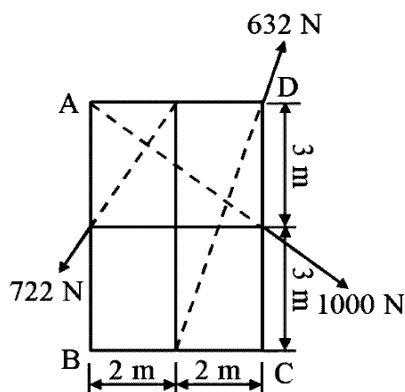
Substituting the values in equation

$$v = 1.5344 \text{ m/s}$$



Velocity of block  $P_1 = 1.5344 \text{ m/s}$  (towards down)

**Q2(a) Compute the resultant of three forces acting on the plate shown in the figure. Locate it's intersection with AB and BC. (6 marks)**

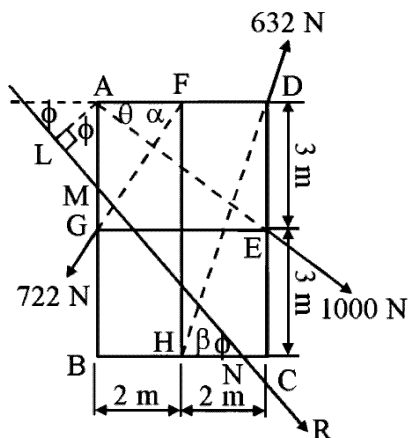


**Solution :**

**Given :** Various forces acting on a body

**To find :** Resultant of the forces and intersection of resultant with AB and BC

**Solution :**



In  $\triangle AFG$ ,

$$\tan \alpha = \frac{AG}{AF} = \frac{DE}{BH} = \frac{3}{2} = 1.5$$

$$\alpha = \tan^{-1}(1.5) = 56.31^\circ$$

In  $\triangle DAE$ ,

$$\tan \theta = \frac{DE}{AD} = \frac{DE}{BC} = \frac{3}{4} = 0.75$$

$$\theta = \tan^{-1}0.75 = 36.87^\circ$$

In  $\triangle DHC$

$$\tan \beta = \frac{DC}{HC} = \frac{6}{2} = 3$$

$$\beta = \tan^{-1}(3)$$

$$\beta = 71.565^\circ$$

Assume R be the resultant of the forces

$$\begin{aligned} \Sigma F_x &= -722 \cos \alpha + 1000 \cos \theta + 632 \cos \beta \\ &= 599.3624 \text{ N} \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= -722 \sin \alpha - 1000 \sin \theta + 632 \sin \beta \\ &= -601.1725 \text{ N} \end{aligned}$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$R = \sqrt{(599.3624)^2 + (-601.1725)^2}$$

$$\mathbf{R=848.9073 \text{ N}}$$

$$\phi = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{-601.1725}{599.3624}\right)$$

$$= 45.0863^\circ \text{ (in fourth quadrant)}$$

**Let R cut AB and BC at points M and N respectively**

Draw  $AL \perp R$

Taking moments about point A

$$\begin{aligned} M_A &= 632 \sin \beta \times AD - 722 \cos \alpha \times AG \\ &= 632 \times \sin 71.565^\circ \times 4 - 722 \cos 56.31^\circ \times 3 \\ &= 1196.7908 \text{ Nm} \end{aligned}$$

**Applying Varignon's theorem**

$$M_A = R \times AL$$

$$1196.7908 = 848.9073 \times AL$$

$$\mathbf{AL = 1.4098 \text{ m}}$$

In  $\triangle AML$ ,

$$\cos \Phi = \frac{AL}{AM}$$

$$\cos 45.0863^\circ = \frac{1.4098}{AM}$$

$$AM = 1.9967 \text{ m}$$

$$MB = AB - AM$$

$$= 6 - 1.9967$$

$$= 4.0033 \text{ m}$$

In  $\triangle BMN$

$$\tan \Phi = \frac{BM}{BN}$$

$$\tan 45.0863^\circ = \frac{4.0033}{BN}$$

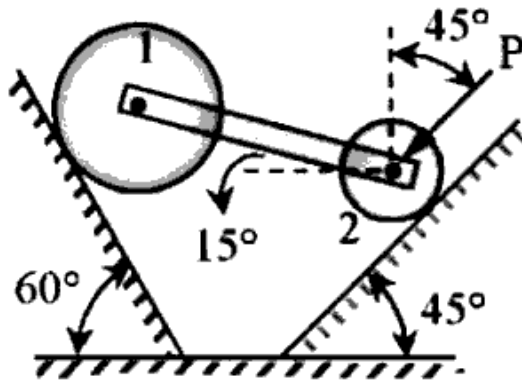
$$\mathbf{BN = 3.9912 \text{ m}}$$

$$\mathbf{R = 848.9073 \text{ N (} 45.0863^\circ \text{ in fourth quadrant)}}$$

**Resultant force intersects AB and BC at M and N such that AM=1.9967 m and BN=3.9912 m**

**Q.2(b)** Two cylinders 1 and 2 are connected by a rigid bar of negligible weight hinged to each cylinder and left to rest in equilibrium in the position shown under the application of force  $P$  applied at the center of cylinder 2.

Determine the magnitude of force  $P$ . If the weights of the cylinders 1 and 2 are 100 N and 50 N respectively. (8 marks)



**Solution :**

**Given :**  $W_1 = 100 \text{ N}$

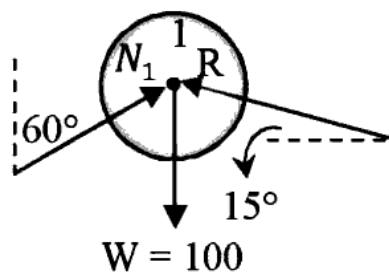
$W_2 = 50 \text{ N}$

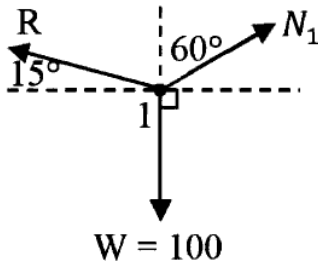
Cylinders are connected by a rigid bar

**To find :** Magnitude of force  $P$

**Solution :**

Consider cylinder I





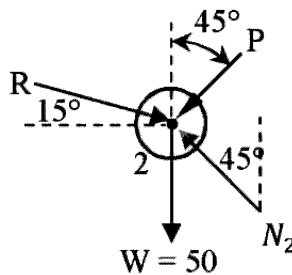
Applying Lami's theorem :

$$\frac{R}{\sin(90+30)} = \frac{W}{\sin(60+75)} = \frac{N_1}{\sin(90+15)}$$

$$R = \frac{100}{\sin 135} \times \sin 120$$

$$R = 122.4745 \text{ N}$$

Cylinder 2 is under equilibrium



Applying conditions of equilibrium

$$\Sigma F_y = 0$$

$$N_2 \sin 45 - R \sin 15 - P \sin 45 - W = 0$$

$$N_2 \sin 45 - P \sin 45 = 122.4745 \times 0.2588 + 50$$

$$N_2 \sin 45 - P \sin 45 = 81.6987 \dots\dots\dots(1)$$

Applying conditions of equilibrium

$$\Sigma F_x = 0$$

$$-N_2 \cos 45 + R \cos 15 - P \cos 45 = 0$$

$$N_2 \cos 45 + P \cos 45 = 118.3013 \quad \dots\dots(2)$$

Solving (1) and (2)

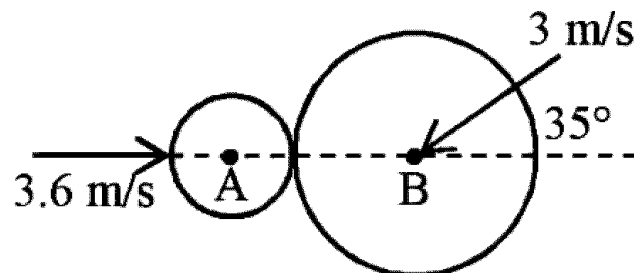
$$P = 25.8819 \text{ N}$$

**Magnitude of force P required = 25.8819 N**

**Q.2(c) Just before they collide, two disk on a horizontal surface have velocities shown in figure.**

**Knowing that 90 N disk A rebounds to the left with a velocity of 1.8 m/s. Determine the rebound velocity of the 135 N disk B. Assume the impact is perfectly elastic.**

**(6 marks)**



**Solution :****Given :**  $W_A = 90\text{N}$ 

$$W_B = 135\text{ N}$$

**Taking velocity direction towards right as positive and towards left as negative**

Initial velocity of disk A = 3.6 m/s

Final velocity of disk A = -1.8 m/s

Initial velocity of disk B = 3 m/s

**To find :** Rebound velocity of disk B**Solution :**

$$m_A = \frac{90}{g} \text{ kg}$$

$$m_B = \frac{135}{g} \text{ kg}$$

Consider the X and Y components of  $u_B$ 

$$u_{BX} = -u_B \cos 35 = -2.4575 \text{ m/s}$$

$$u_{BY} = -u_B \sin 35 = -1.7207 \text{ m/s}$$

**APPLYING LAW OF CONSERVATION OF MOMENTUM :**

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$\frac{90}{g} \times 3.6 + \frac{135}{g} \times (-2.4575) = \frac{90}{g} \times (-1.8) + \frac{135}{g} \times v_{BX}$$

$$v_{BX} = 1.1425 \text{ m/s}$$

**As the impact takes place along X-axis, the velocities of two disks remains same along Y-axis**

$$v_{BY} = u_{BY} = -1.7207 \text{ m/s}$$

$$v = \sqrt{(v_{BX})^2 + (v_{BY})^2}$$

$$v = \sqrt{1.1425^2 + (-1.7207)^2}$$

$$v = 2.0655 \text{ m/s}$$

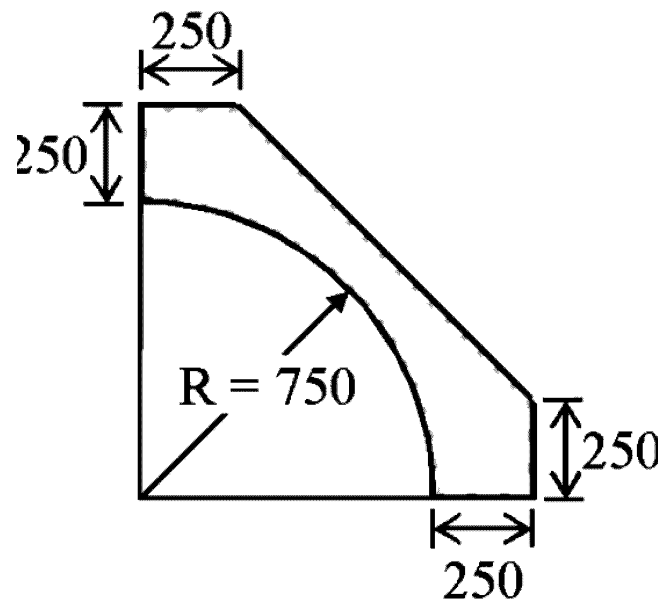
$$\alpha = \tan^{-1}\left(\frac{-1.7207}{1.1425}\right)$$

$$\alpha = 56.4169^\circ$$

**VELOCITY OF DISK B AFTER IMPACT = 2.0655 m/s (56.4169° in fourth quadrant)**

**Q.3(a) Find the centroid of the shaded portion of the plate shown in the figure.**

**(8 marks)**



**Solution :**

$Y = X$  is the axis of symmetry

The centroid would lie on this line

Sr.no.	PART	AREA(in mm <sup>2</sup> )	X co-ordinate(mm)	Ax(mm <sup>3</sup> )



1.	RECTANGLE	=1000 X 1000 =1000000	$\frac{1000}{2} = 500$	500000000
2.	TRIANGLE (to be removed)	$\frac{1}{2} \times 750 \times 750$ = -281250	$1000 - \frac{750}{3}$ = 750	-210937500
3.	QUARTER CIRCLE (To be removed)	$\frac{\pi r^2}{4}$ = 441786.4669	$\frac{4 \times 750}{3\pi}$ = 3141.5926	-140625000
	TOTAL	276963.4669		148437500

$$\bar{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{148437500}{276963.5331} = 535.946 \text{ mm}$$

$$\bar{y} = \bar{X} = 535.946 \text{ mm}$$

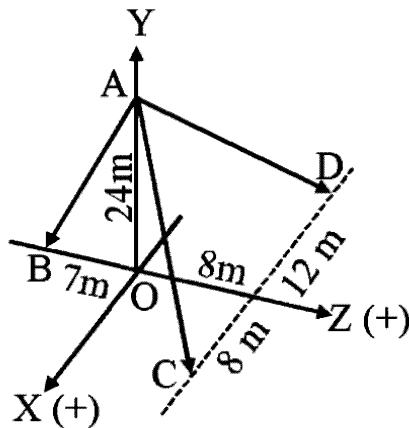
**CENTROID IS AT (535.946,535.946)mm**

**Q.3(b) Co-ordinate distance are in m units for the space frame in figure.**

**There are 3 members AB,AC and AD. There is a force W=10 kN acting at A in a vertically upward direction.**

**Determine the tension in AB,AC and AD.**

**(6 marks)**



**Solution :**

**Given :**  $A = (0,24,0)$

$$B = (0,0,-7)$$

$$C = (8,0,8)$$

$$D = (-12,0,8)$$

**To find :** Tension in AB, AC and AD.

**Solution :**

Assume  $\bar{a}, \bar{b}, \bar{c}, \bar{d}$  be the position vectors of points A, B, C, D with respect to origin O.

$$\overline{OA} = \bar{a} = 24\bar{j}$$

$$\overline{OB} = \bar{b} = -7\bar{k}$$

$$\overline{OC} = \bar{c} = 8\bar{i} + 8\bar{k}$$

$$\overline{OD} = \bar{d} = -12\bar{i} + 8\bar{k}$$

$$\overline{AB} = \bar{b} - \bar{a} = -24\bar{j} - 7\bar{k} \quad \text{Magnitude} = 25$$

$$\text{Unit vector} = \frac{-24\bar{j} - 7\bar{k}}{25}$$

$$\overline{AC} = \bar{c} - \bar{a} = 8(\bar{i} - 3\bar{j} + \bar{k}) \quad \text{Magnitude} = 8\sqrt{11}$$

$$\text{Unit vector} = \frac{8(\bar{i} - 3\bar{j} + \bar{k})}{8\sqrt{11}}$$

$$\overline{AD} = \bar{d} - \bar{a} = 4(-3\bar{i} - 6\bar{j} + 2\bar{k}) \quad \text{Magnitude} = 28$$

$$\text{Unit vector} = \frac{4(-3\bar{i} - 6\bar{j} + 2\bar{k})}{28}$$

Assume  $T_1, T_2$  and  $T_3$  be the tensions along AB, AC and AD

$$\mathbf{T}_1 = T_1 \left( \frac{-24\bar{j} - 7\bar{k}}{25} \right)$$

$$\mathbf{T}_2 = T_2 \left( \frac{8(\bar{i} - 3\bar{j} + \bar{k})}{8\sqrt{11}} \right)$$

$$\mathbf{T}_3 = T_3 \left( \frac{4(-3\bar{i} - 6\bar{j} + 2\bar{k})}{28} \right)$$

A force of 10kN is acting at point A in vertically upward direction

Applying conditions of equilibrium

$$10\bar{j} + \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = \mathbf{0}$$

$$-10\bar{j} = T_1\left(\frac{-24\bar{j}-7\bar{k}}{25}\right) + T_2\left(\frac{8(i-3j+k)}{8\sqrt{11}}\right) + T_3\left(\frac{4(-3i-6j+2k)}{28}\right)$$

$$0\bar{i} - 10\bar{j} + 0\bar{k} = T_1\left(\frac{-24\bar{j}-7\bar{k}}{25}\right) + T_2\left(\frac{8(i-3j+k)}{8\sqrt{11}}\right) + T_3\left(\frac{4(-3i-6j+2k)}{28}\right)$$

**Comparing both sides of equation**

$$\frac{T_2}{\sqrt{11}} - \frac{3T_3}{7} = 0$$

$$\frac{-24T_1}{25} - \frac{3T_2}{\sqrt{11}} - \frac{6T_3}{7} = -10$$

$$\frac{-7T_1}{25} - \frac{T_2}{\sqrt{11}} + \frac{2T_3}{7} = 0$$

Solving the equations simultaneously

$$\mathbf{T_1=5.5556 \text{ N}}$$

$$\mathbf{T_2=3.0955 \text{ N}}$$

$$\mathbf{T_3=2.1778}$$

$$\mathbf{T_{AB} = -5.3333\bar{j} - 1.5556\bar{k}}$$

$$\mathbf{T_{AC} = 0.9333\bar{i} - 2.8\bar{j} + 0.9333\bar{k}}$$

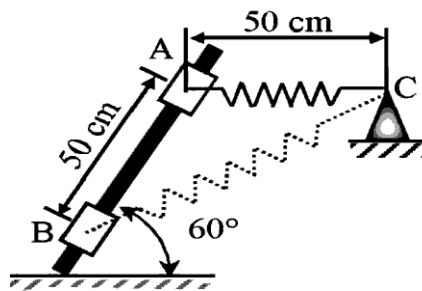
$$\mathbf{T_{AD} = -0.9333\bar{i} - 1.8667\bar{j} + 0.6222\bar{k}}$$


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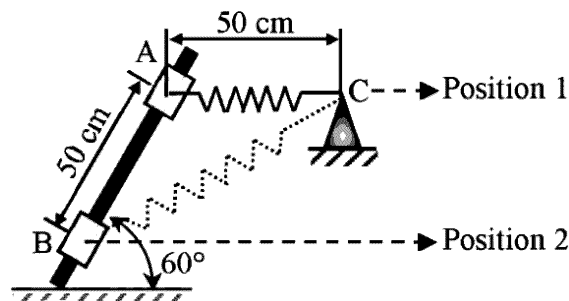
**Q.3(c) A 50 N collar slides without friction along a smooth rod which is kept inclined at  $60^\circ$  to the horizontal.**

**The spring attached to the collar and the support C. The spring is unstretched when the roller is at A (AC is horizontal).**

**Determine the value of spring constant  $k$  given that the collar has a velocity of 2.5 m/s when it has moved 0.5 m along the rod as shown in the figure. (6 marks)**



**Solution :**



**Given :**  $W=50\text{ N}$

$$AB = AC = 0.5\text{ m}$$

**To find :** Spring constant

**Solution :**

$$\text{Mass of collar} = \frac{50}{g} \text{ kg}$$

Let us assume that  $h = 0$  at position 2

**POSITION 1 :**

$$x = 0$$

$$E_{s1} = \frac{1}{2} k x x_1^2 = 0$$

$$h_1 = 0.5 \sin 60 = 0.433 \text{ m}$$

$$PE_1 = mgh_1 = 21.65 \text{ J}$$

$$v_A = 0 \text{ m/s}$$

$$KE_1 = 0 \text{ J}$$

**POSITION II :**

$$v_B = 2.5 \text{ m/s}$$

$$PE_2 = mgh = 0 \text{ J (because } h=0)$$

$$\begin{aligned} KE_2 &= \frac{1}{2} X m v^2 = \frac{1}{2} X \frac{50}{g} X 2.5^2 \\ &= 15.9276 \text{ J} \end{aligned}$$

In  $\triangle ABC$

Applying cosine rule

$$\begin{aligned} BC^2 &= AB^2 + AC^2 - 2 X AB X AC X \cos(BAC) \\ &= 0.5^2 + 0.5^2 - 2 x 0.5 x 0.5 x \cos 120 \\ &= 0.75 \end{aligned}$$

$$BC = 0.866 \text{ m}$$

Un-stretched length of the spring = 0.5 m

Extension of spring(x) = 0.866 - 0.5

$$= 0.366 \text{ m}$$

$$\begin{aligned} E_{s2} &= \frac{1}{2} x k x_2^2 \\ &= 0.067k \end{aligned}$$

APPLYING WORK ENERGY PRINCIPLE

$$U_{1-2} = KE_2 - KE_1$$

$$PE_1 - PE_2 + ES_1 - ES_2 = KE_2 - KE_1$$

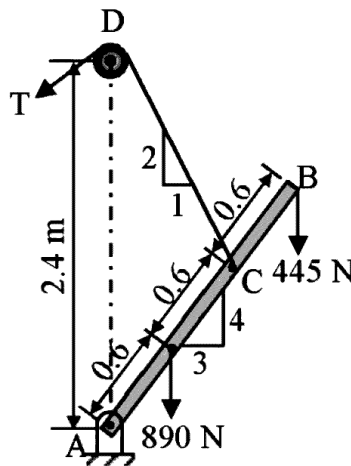
$$21.6506 - 0 + 0 - 0.067K = 15.9276 - 0$$

$$K = 85.4343 \text{ N/m}$$

**SPRING CONSTANT IS 85.4343 N/m**

**Q.4(a) A boom AB is supported as shown in the figure by a cable runs from C over a small smooth pulley at D.**

**Compute the tension T in cable and reaction at A. Neglect the weight of the boom and size of the pulley. (8 marks)**



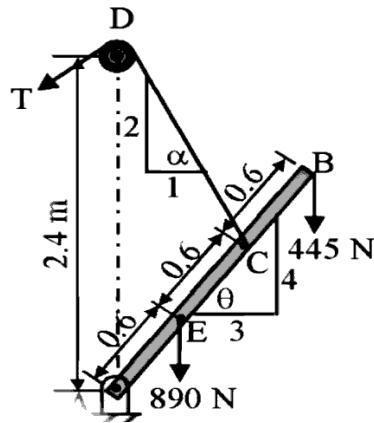
**Solution :**

**Given :** Beam AB is supported by a cable

**To find :** Tension T in cable

Reaction at A

**Solution :**



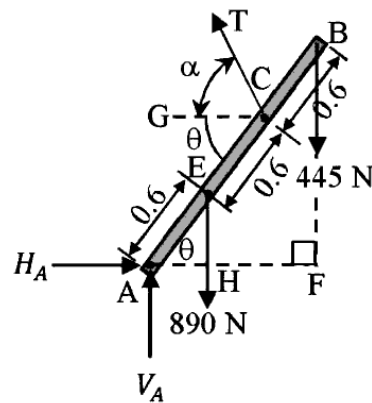
$$\tan \alpha = \frac{2}{1}$$

$$\alpha = 63.4349^\circ$$

$$\tan \theta = \frac{4}{3}$$

$$\theta = 53.13^\circ$$

Assume  $H_A$  and  $V_A$  be the horizontal and vertical reaction forces at A



$$\angle GCA = \angle BAF = \theta$$

$$\angle TCG = \alpha$$

$$\angle TCA = \alpha + \theta$$

$$= 63.4349^\circ + 53.16^\circ$$

$$= 116.5651^\circ$$

$$\angle TCB = 180^\circ - 116.5651^\circ$$

$$= 63.4349^\circ$$

$$AC = AE + EC = 0.6 + 0.6 = 1.2$$

$$AB = AC + CB = 1.2 + 0.6 = 1.8$$

$$AF = AB \cos \theta = 1.8 \cos 53.13 = 1.08$$

$$AH = AE \cos \theta = 0.6 \cos 53.13 = 0.36$$

### BEAM AB IS IN EQUILIBRIUM

Applying conditions of equilibrium

$$\Sigma M_A = 0$$

$$-445 \times AF - 890 \times AH + T \sin 63.4349 \times AC = 0$$

$$T \times 0.8944 \times 1.2 = 445 \times 1.08 + 890 \times 0.36$$

$$T = 746.2877 \text{ N}$$

$$\Sigma F_x = 0$$

$$H_A - T \cos 63.4349 = 0$$

$$H_A = 333.75 \text{ N}$$

$$\Sigma F_y = 0$$

$$V_A + T \sin 63.4349 - 890 - 445 = 0$$

$$V_A = 667.5 \text{ N}$$

$$R_A = \sqrt{H_A^2 + V_A^2}$$

$$R_A = \sqrt{(333.75)^2 + (667.5)^2}$$

$$R_A = 746.2877 \text{ N}$$

$$\Phi = \tan^{-1}\left(\frac{V_A}{H_A}\right)$$

$$\Phi = \tan^{-1}\left(\frac{667.5}{333.75}\right)$$



$$\Phi = 63.4395^\circ$$

**Tension in cable = 746.2877 N (63.43949° in second quadrant)**

**Reaction at A = 746.2877 N (63.4395° in first quadrant)**

**Q.4(b) The acceleration of the train starting from rest at any instant is given by the expression  $a = \frac{8}{v^2 + 1}$  where  $v$  is the velocity of train in m/s.**

**Find the velocity of the train when its displacement is 20 m and its displacement when velocity is 64.8 kmph. (6 marks)**

**Solution :**

**Given :**  $a = \frac{8}{v^2 + 1}$

**To find :** Velocity when displacement is 20 m

Displacement when velocity is 64.8 kmph.

**Solution :**

$$a = v \frac{dv}{dx}$$

$$v \frac{dv}{dx} = \frac{8}{v^2 + 1}$$

$$v(v^2 + 1)dv = 8dx$$

Integrating both sides

$$\int v(v^2 + 1)dv = \int 8dx$$

$$\frac{v^4}{4} + \frac{v^2}{2} = 8x + c \quad \dots\dots\dots(1)$$

Multiplying by 4 on both sides

$$V^4 + 2v^2 = 32x + 4c$$

Substituting  $v=0$  and  $x=0$  in (1)

$$c=0$$

From (1)

$$V^4 + 2v^2 = 32x \quad \dots\dots\dots(2)$$

**Case 1 : x=20 m**

$$V^4 + 2v^2 = 32 \times 20 \quad \dots\dots\dots(\text{From 2})$$

$$V^4 + 2v^2 - 640 = 0$$

Solving the equation

$$V^2 = 24.3180$$

$$V = 4.9361 \text{ m/s}$$

**Case 2 : V=64.8 kmph(or v = 18 m/s)**

$$18^4 + 2 \times 18^2 = 32x \quad \dots\dots\dots(\text{From 2})$$

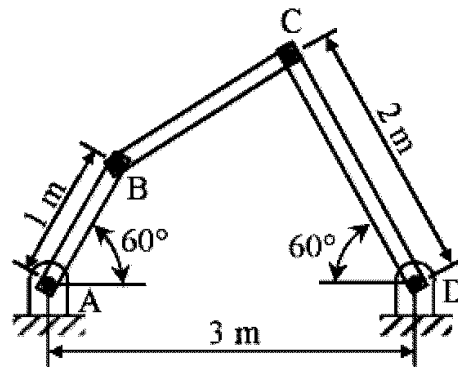
$$1.5624 = 32x$$

$$x = 3300.75 \text{ m}$$

**When displacement of train is 20 m, then velocity is 4.9361 m/s**

**When velocity of the train is 64.8 kmph, then its displacement is 3300.75m**

**Q.4(c) Angular velocity of connector BC is 4 r/s in clockwise direction. What is the angular velocities of cranks AB and CD? (6 marks)**



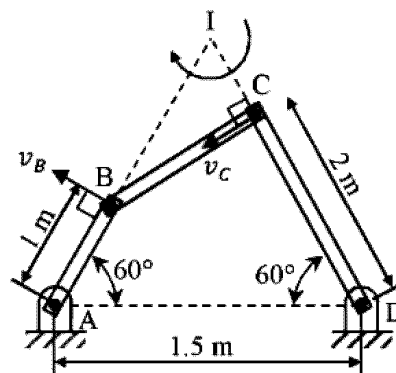
**Solution:**

**Given :** Angular velocity of BC is 4 rad/s

**To find :** Angular velocity of AB and CD

**Solution:**

ICR is shown in the figure



**USING GEOMETRY :**

In  $\triangle IAD$

$$\angle A = \angle D = 60^\circ$$

$$\angle I = 60^\circ$$

**Δ IAD is equilateral**

$$IA = ID = AD = 3 \text{ cm}$$

$$IB + AB = IA$$

$$\mathbf{IB = 2 \text{ cm}}$$

Similarly, we can solve that  $IC = 1 \text{ cm}$

$$\mathbf{v = r\omega}$$

$$v_B = IB \times \omega_{BC} = 8 \text{ m/s}$$

$$v_C = IC \times \omega_{BC} = 4 \text{ m/s}$$

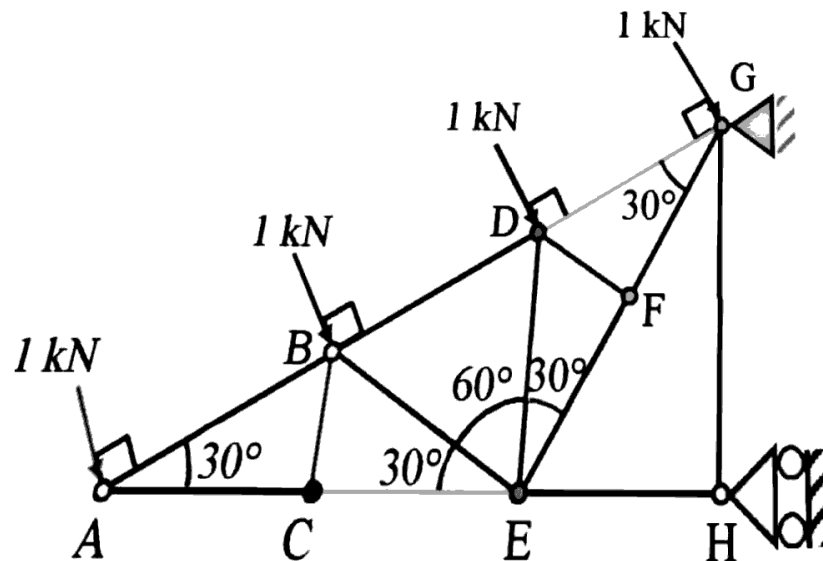
$$\omega_{AB} = \frac{v_B}{AB} = \frac{8}{1} = 8 \text{ rad/s (Anti-clockwise)}$$

$$\omega_{DC} = \frac{v_C}{DC} = \frac{4}{2} = 2 \text{ rad/s (Anti-clockwise)}$$

**Angular velocity of AB = 8 rad/s (Anti-clockwise)**

**Angular velocity of CD = 2 rad/s (Anti-clockwise)**

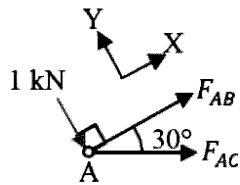
**Q.5(a) In the truss shown in figure, compute the forces in each member. (8 marks)**



**Solution :**

We can say that FD, GH and CB are zero force members in the given truss

**Joint A :**



Applying the conditions of equilibrium

$$\Sigma F_y = 0$$

$$-1 - F_{AC} \sin 30 = 0$$

$$F_{AC} = -2 \text{ kN}$$

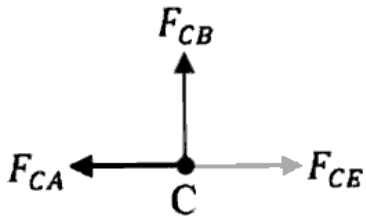
Applying the conditions of equilibrium

$$\Sigma F_x = 0$$

$$F_{AB} + F_{AC} \cos 30 = 0$$

$$F_{AB} = 1.7321 \text{ Kn}$$

**JOINT C :**

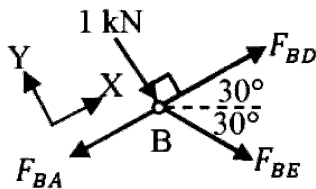


Applying the conditions of equilibrium

$$\Sigma F_x = 0$$

$$F_{CE} = F_{CA} = -2\text{kN}$$

**JOINT B :**



Applying the conditions of equilibrium

$$\Sigma F_y = 0$$

$$-1 - F_{BE} \sin 60 = 0$$

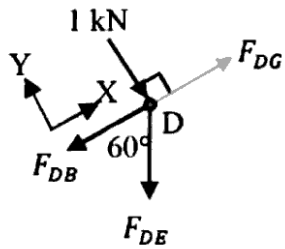
$$F_{BE} = -1.1547 \text{ kN}$$

Applying the conditions of equilibrium

$$\Sigma F_x = 0$$

$$-F_{BA} + F_{BE} \cos 60 + F_{BD} = 0$$

$$F_{BD} = 2.3094 \text{ kN}$$

**JOINT D :**

Applying the conditions of equilibrium

$$\Sigma F_y = 0$$

$$-1 - F_{DE}\sin 60 = 0$$

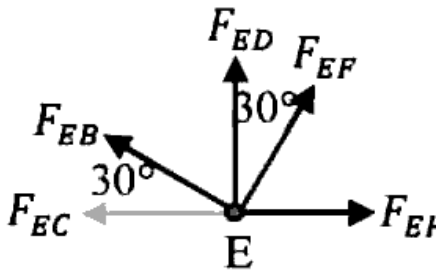
$$F_{DE} = -1.1547 \text{ kN}$$

Applying the conditions of equilibrium

$$\Sigma F_x = 0$$

$$-F_{DB} - F_{DE}\cos 60 + F_{DG} = 0$$

$$F_{DG} = 1.7321 \text{ kN}$$

**JOINT E :**

Applying the conditions of equilibrium

$$\Sigma F_y = 0$$

$$F_{ED} + F_{EF}\cos 30 + F_{EB}\sin 30 = 0$$

$$F_{EF}\cos 30 = -(-1.1547) - (-1.1547) \times \frac{1}{2}$$

$$F_{EF} = 2 \text{ kN}$$

Applying the conditions of equilibrium

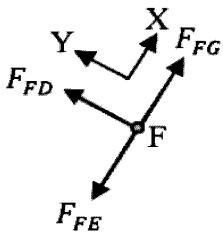
$$\Sigma F_x = 0$$

$$-F_{EC} + F_{EH} + F_{EF}\sin 30 - F_{EB}\cos 30 = 0$$

$$F_{EH} = F_{EC} - F_{EF}\sin 30 + F_{EB}\cos 30$$

$$F_{EH} = -4\text{kN}$$

**Joint F :**



Applying the conditions of equilibrium

$$\Sigma F_x = 0$$

$$F_{FG} = F_{FE} = -2\text{kN}$$

**Final answer :**

Sr.no.	MEMBER	MAGNITUDE OF FORCE (in kN)	NATURE OF FORCE
1.	AC	2	COMPRESSION
2.	AB	1.7321	TENSION
3.	CB	0	-
4.	CE	2	COMPRESSION
5.	BE	1.1547	COMPRESSION
6.	BD	2.3094	TENSION
7.	DE	1.1547	COMPRESSION

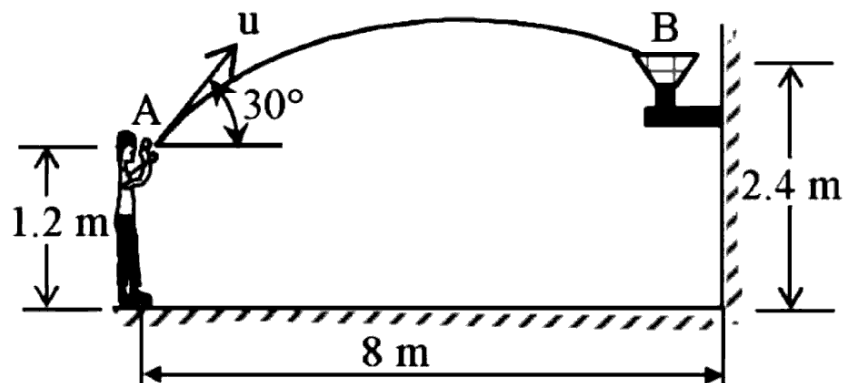


8.	DG	1.7321	TENSION
9.	EF	2	TENSION
10.	EH	4	COMPRESSION
11.	FD	0	-
12.	FG	2	COMPRESSION
13.	GH	0	-

**Q.5(b) Determine the speed at which the basket ball at A must be thrown at an angle  $30^\circ$  so that it makes it to the basket at B.**

**Also find at what speed it passes through the hoop.**

**(6 marks)**



**Solution :**

**Given :**  $\theta=30^\circ$

**To find :** Speed at which basket ball must be thrown

**Solution :**

Assume that the basket ball be thrown with initial velocity  $u$  and it takes time  $t$  to reach B

**HORIZONTAL MOTION**

Here the velocity is constant

$$8 = u \cos 30 \times t$$

$$t = \frac{8}{u \cos 30} = \frac{9.2376}{u} \dots\dots(1)$$

$$v_B = u \cos 30 \quad (\text{Since velocity is constant in horizontal motion}) \dots\dots(2)$$

**VERTICAL MOTION**

$$\text{Initial vertical velocity } (u_v) = u \sin 30 = 0.5u \dots\dots(3)$$

$$\text{Vertical displacement}(s) = 2.4 - 1.2 = 1.2$$

$$t = \frac{9.2376}{u}$$

Using kinematical equation :

$$s = ut + \frac{1}{2} \times at^2$$

$$1.2 = \frac{u}{2} \times \frac{9.2376}{u} - \frac{1}{2} \times 9.81 \times \left(\frac{9.2376}{u}\right)^2$$

$$u^2 = 122.4289$$

$$\mathbf{u = 11.0648 \text{ m/s}}$$

$$u_v = 0.5u \quad (\text{From 3})$$

$$u_v = 0.5 \times 11.0648$$

$$= 5.5324 \text{ m/s}$$

Using kinematical equation

$$v_v^2 = u_v^2 + 2as$$

$$v_v^2 = 5.5324^2 - 2 \times 9.81 \times 1.2$$

$$\mathbf{v_v = 2.6622 \text{ m/s}}$$

$$v_h = 11.0648 \cos 30 = 9.5824 \text{ m/s} \quad (\text{From 2})$$

$$v_B = \sqrt{v_v^2 + v_h^2}$$

$$\mathbf{v_B = 9.9441 \text{ m/s}}$$

$$\alpha = \tan^{-1}\left(\frac{2.6577}{9.5824}\right)$$

$$= 15.5015^\circ$$

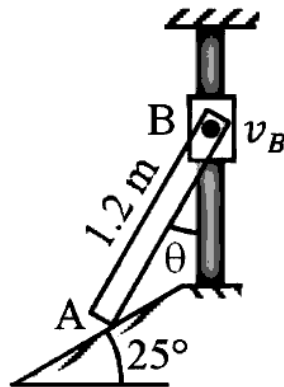
**Speed at which the basket-ball at A must be thrown = 11.0648 m/s (30° in first quadrant)**

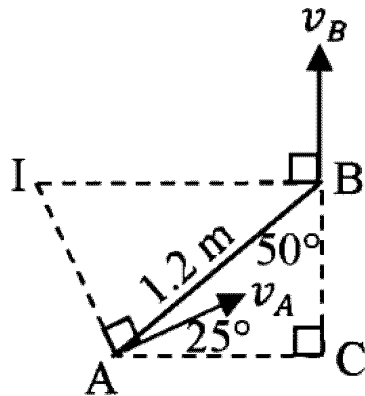
**Speed at which the basket-ball passes through the hoop = 9.9441 m/s(15.5015° in fourth quadrant)**

**Q.5(c) Figure shows a collar B which moves upwards with constant velocity of 1.5 m/s. At the instant when  $\theta=50^\circ$ . Determine :**

**(i) The angular velocity of rod pinned at B and freely resting at A against 25° sloping ground.**

**(ii) The velocity of end A of the rod. (6 marks)**



**Solution:**

ICR is shown in the given figure

**BY USING GEOMETRY:**

In  $\triangle ABC$

$$\angle ABC = 50$$

$$\angle ACB = 90$$

$$\angle BAC = 40$$

$$\angle CAV = 25$$

$$\angle BAV = 40 - 25 = 15$$

$IA \perp V_A$

$$\angle IAB = 90 - 15 = 75$$

$$\angle IBA = 90 - 50 = 40$$

In  $\triangle IBA$

$$\angle BIA = 180 - 75 = 65$$

In  $\triangle IBA$

$$AB = 1.2 \text{ m}$$

**APPLYING SINE RULE**

$$\frac{AB}{\sin I} = \frac{IB}{\sin A} = \frac{IA}{\sin B}$$

$$\frac{1.2}{\sin 65} = \frac{IB}{\sin 75} = \frac{IA}{\sin 40}$$

$$IB=1.2789 \text{ m}$$

$$IA=0.8511 \text{ m}$$

Assume  $\omega_{AB}$  be the angular velocity of AB

$$\omega_{AB} = \frac{v_B}{r} = \frac{v_B}{IB} = \frac{1.5}{1.2789} = 1.1728 \text{ rad/s}$$

$$v_A = r \times \omega_{AB} = IA \times \omega_{AB} = 0.8511 \times 1.1728 = 0.99825 \text{ m/s}$$

**Angular velocity of rod AB= 1.1728 rads (Anti-clockwise)**

**Instantaneous velocity of A = 0.9982 m/s( 25° in first quadrant)**

**Q.6(a) A force of 140 kN passes through point C (-6,2,2) and goes to point B (6,6,8).**

**Calculate moment of force about origin.**

**(4 marks)**

**Solution :**

**Given :** C (-6,2,2)

B (6,6,8)

**To find :** Moment of force about origin

**Solution :**

Assume  $\bar{b}$  and  $\bar{c}$  be the position vectors of points B and C respectively w.r.t O (0,0,0)

$$\overline{OB} = \bar{b} = 6\bar{i} + 6\bar{j} + 8\bar{k}$$

$$\overline{OC} = -6\bar{i} + 2\bar{j} + 2\bar{k}$$

$$\overline{CB} = (6\bar{i} + 6\bar{j} + 8\bar{k}) - (-6\bar{i} + 2\bar{j} + 2\bar{k})$$

$$= 2(6\bar{i} + 2\bar{j} + 3\bar{k})$$

$$|\overline{CB}| = 2 \times \sqrt{6^2 + 2^2 + 3^2}$$

$$= 14$$

$$\text{Unit vector along } \overline{CB} = \frac{\overline{CB}}{|\overline{CB}|} = \frac{6\bar{i} + 2\bar{j} + 3\bar{k}}{7}$$

$$\begin{aligned}\text{Force along } \overline{CB} = \overline{F} &= 140 \times \frac{6i+2j+3k}{7} \\ &= 120\overline{i} + 40\overline{j} + 60\overline{k}\end{aligned}$$

$$\text{Moment of } \overline{F} \text{ about O} = \overline{OB} \times \overline{F}$$

$$\begin{array}{ccc} i & j & k \\ 6 & 6 & 8 \\ 120 & 40 & 60 \end{array}$$

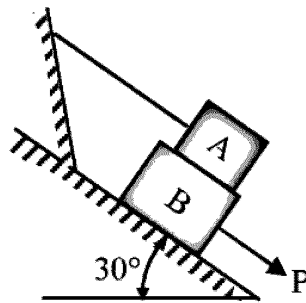
$$= 40\overline{i} + 600\overline{j} - 480\overline{k}$$

**Moment of F about C is  $40\overline{i} + 600\overline{j} - 480\overline{k}$  kNm**

**Q.6(b) Refer to figure. If the co-efficient of friction is 0.60 for all contact surfaces and  $\theta = 30^\circ$ , what force P applied to the block B acting down and parallel to the incline will start motion and what will be the tension in the cord parallel to inclined plane attached to A.**

**Take  $W_A = 120$  N and  $W_B = 200$  N.**

**(8 marks)**



**Solution :**

**Given :**  $\mu = 0.6$

$$\theta = 30^\circ$$

$$W_A = 120 \text{ N}$$

$$W_B = 200 \text{ N}$$

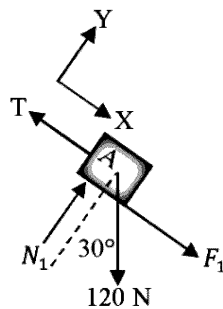
**To find :** Force P

**Solution :**

$$F_1 = \mu N_1 = 0.6N_1 \quad \dots\dots\dots(1)$$

$$F_2 = \mu N_2 = 0.6N_2 \quad \dots\dots\dots(2)$$

**Consider FBD of block A**



The block is considered to be in equilibrium

**Applying conditions of equilibrium**

$$\Sigma F_y = 0$$

$$N_1 - 120\cos 30 = 0$$

$$N_1 = 103.923 \text{ N} \quad \dots\dots\dots(3)$$

From (1)

$$F_1 = 0.6 \times 103.923$$

$$= 62.3538 \text{ N}$$

**Applying conditions of equilibrium**

$$\Sigma F_x = 0$$

$$F_1 + 120\sin 30 - T = 0$$

$$T = 122.3538 \text{ N}$$

Consider FBD of block B

Applying conditions of equilibrium

$$\Sigma F_y = 0$$

$$N_2 - N_1 - 200\cos 30 = 0$$

$$N_2 = 277.1281 \text{ N}$$

$$F_2 = 0.6 \times 277.1281$$

$$= 166.2769 \text{ N} \quad \text{From (2)}$$

Applying conditions of equilibrium

$$\Sigma F_x = 0$$

$$P - F_1 - F_2 + 200\sin 30 = 0$$

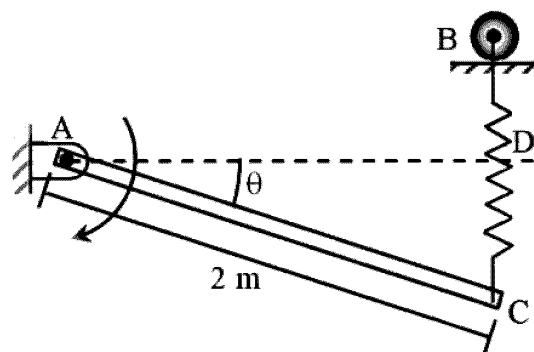
$$P = 128.6307 \text{ N}$$

**Force required on block B to start the motion is 128.6307 N**

**Tension T in the cord parallel to inclined plane attached to A=122.3538 N**

**Q.6(c) Determine the required stiffness k so that the uniform 7 kg bar AC is in equilibrium when  $\theta=30^\circ$ .**

**Due to the collar guide at B the spring remains vertical and is unstretched when  $\theta = 0^\circ$ . Use principle of virtual work. (4 marks)**





**Solution:****Given :** Mass of bar AC = 7 kg

$$\theta = 30^\circ$$

**To find :** Required stiffness k**Solution:**

Weight of rod = 7g N

Assume rod AC have a small virtual angular displacement  $\delta\theta$  in anti-clockwise direction**Reaction forces  $H_A$  and  $V_A$  do not do any virtual work**

Un-stretched length of the spring = BD

Extension of the spring (x) = CD = 2sin  $\theta$ Assume  $F_s$  be the spring force at end C of the rod

$$F_s = Kx = 2K\sin \theta$$

Assume A to be the origin and AD be the X-axis of the system

Active force	Co-ordinate of the point of action along the force	Virtual Displacement
$W=7g$	$-\sin \theta$	$\delta y_M = -\cos \theta \delta \theta$
$F_s=2K\sin \theta$	$-2\sin \theta$	$\delta y_{C'} = -2\cos \theta \delta \theta$

**APPLYING PRINCIPLE OF VIRTUAL WORK**

$$\delta U = 0$$

$$-W \times \delta y_M + F_s \times \delta y_{C'} + 50 \times \delta \theta = 0$$

$$2K\sin \theta \times (-2\cos \theta \delta \theta) = 7g \times (-\cos \theta \delta \theta) - 50 \times \delta \theta$$

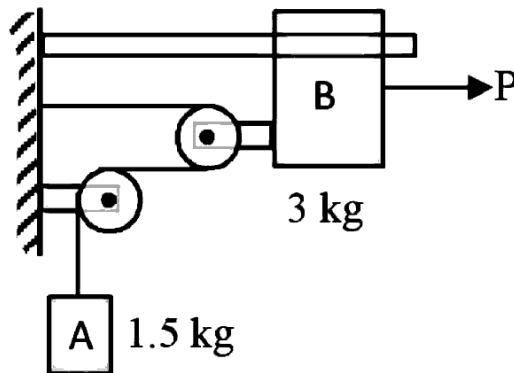
Substituting the value of  $\theta$  and solving

$$K=63.2025 \text{ Nm}$$

**The required stiffness K for bar AC to remain in equilibrium is 63.2025 Nm**

**Q.6(d) The system in figure is initially at rest.**

**Neglecting friction determine the force P required if the velocity of the collar is 5 m/s after 2 sec and corresponding tension in the cable. (4 marks)**



**Solution :**

For block B

$$u = 0$$

$$t = 2 \text{ s}$$

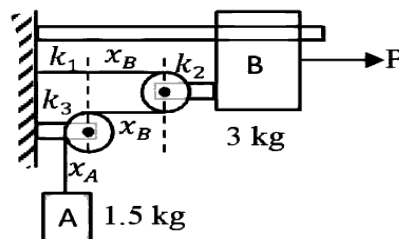
$$v = 5 \text{ m/s}$$

$$a = \frac{5-0}{2} = 2.5 \text{ m/s}^2 \quad \dots\dots\dots(1)$$

Assume the string across the two pulleys be of length L

Assume  $x_A$  and  $x_B$  be the displacements of block A and collar B respectively

Assume  $k_1, k_2$  and  $k_3$  be the lengths of the string which remain constant irrespective of the position of block A and block B



$$k_1 + x_B + k_2 + x_B + k_3 + x_A = L$$

$$x_A = L - k_1 - k_2 - k_3 - 2x_B$$

Differentiating with respect to time

$$v_A = -2v_B$$

Differentiating with respect to time one again

$$a_A = -2a_B$$

Considering only magnitude

$$a_A = 2a_B$$

$$a_A = 2 \times 2.5$$

$$= 5 \text{ m/s}^2 \quad \dots\dots\dots(2) \text{ (From 1)}$$

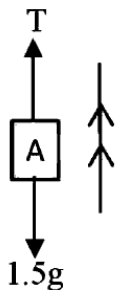
$$\text{Weight of block A}(W_A) = m_A g$$

$$= 14.715 \text{ N}$$

Assume T to be the tension in the string

Consider the vertical motion of block A

F.B.D of block A



$$\Sigma F_y = m_{AA} a_A$$

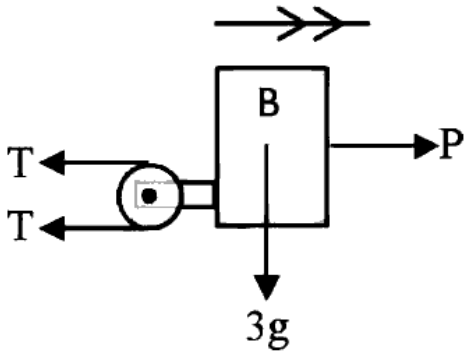
$$T - W_A = m_{AA} a_A$$

$$T - 14.715 = 1.5 \times 5$$

$$T = 22.215 \text{ N} \quad \dots\dots\dots(3)$$

Consider the horizontal motion of collar B

F.B.D of collar B



$$\Sigma F_x = m_B a_B$$

$$P - 2T = m_B a_B$$

$$P - 2 \times 22.215 = 3 \times 2.5$$

$$P = 51.93 \text{ N}$$

**Force P required = 51.93 N**

**Tension in the cable = 22.215 N**