

~~ME-II/SSA EXT C.~~

QP Code : 30052

Duration 03 Hours

Total Marks assigned to the paper 80

Marks assigned to each question should be stated against each question.

Instructions to the candidates, if any:-

N.B.:

- 1) Attempt any Four questions from the Six questions
- 2) Assumptions made should be clearly stated.
- 3) Figures to the right indicate full marks.
- 4) Illustrate answer with sketches wherever required.
- 5) Use of Normal table is permitted.

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- 1 (a) If  $X_1, X_2, \dots, X_n$  are the Poisson variates with parameter  $\lambda = 2$ , use the central limit theorem to estimate  $P(120 \leq S_n \leq 160)$  where  $S_n = X_1 + X_2 + \dots + X_n$  and  $n = 75$ . 10
  - (b) Define random process and give a detailed classification of random process with examples of discrete and continuous random process. 10
  - 2 (a) Let  $X = N(\mu; \sigma^2)$ . Find  $\mu_X$  and  $\sigma_X^2$  10
  - (b) Consider the random process  $X(t)$  defined by 10
$$X(t) = Y \cos(\omega t) \quad t \geq 0$$
where  $\omega$  is a constant and  $Y$  is a uniform r.v. over  $(0, 1)$ .
    - i. Find  $E(X(t))$
    - ii. Find the autocorrelation function of  $X(t)$ .
    - iii. Find the autocovariance function of  $X(t)$ .
  - 3 (a) Let  $X(t) = a \cos(2\pi f_0 t + \Theta)$  where  $\Theta$  is uniformly distributed in the interval  $(0, 2\pi)$ . Find  $S_X(f)$ . 10
  - (b) Write a detailed note on Kalman filter. 10
  - 4 (a) The time elapsed between the claims processed is modeled such that  $T_k$  represents the time elapsed between processing the  $(k-1)^{\text{th}}$  and  $k^{\text{th}}$  claim where  $T_1$  is the time until the first claim is processed, etc. 10

You are given:

    - I.  $T_1, T_2, \dots$  are mutually independent; and
    - II. The pdf of each  $T_k$  is  $f(t) = 0.1 e^{-0.1t}$ , for  $t > 0$  where  $t$  is measured in half-hours.
    - i. Calculate the probability that at least one claim will be processed in the next 5 hrs?
    - ii. What is the probability that at least 3 claims processed within 5 hrs?
  - (b) Find the optimum causal filter for estimating a signal  $Z(t)$  from the observation 10
$$X(t) = Z(t) + N(t)$$

where  $Z(t)$  and  $N(t)$  are independent random processes,  $N(t)$  is a zero-mean white noise with noise density 1 and  $Z(t)$  has power spectral density

$$S_Z(f) = 2/(1 + 4\pi^2 f^2).$$

Find the Wiener optimum filter.

- 5 (a) Describe each of the following random walks with corresponding transition matrix: 10  
General 1-D random walk, random walk with absorbing barriers, random walk with reflecting barriers, and cyclic random walk.
- (b) State and explain Bayes' theorem. 05
- (c) Give the classification of Markov states. 05
- 6 (a) Explain the concept of a typical queueing system with a suitable block diagram. 05
- (b) State and explain Little's formula. 05
- (c) Explain in detail M/M/1 queueing system. 10

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